OFDM-Based Cognitive Radio Networks

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Outline

Spectrum Under-utilization

Based on the reports:

- measurements in US show 70% of the allocated spectrum is not utilized
- some frequency bands are largely unoccupied
- some other frequency bands are only partially occupied
- the remaining frequency bands are heavily used

*Measured Spectrum Occupancy in Chicago

Band	MHz	Occupancy ratio (%)
Fixed Mobile, Amateur, others	138-174	35
TV 14-20	470-512	60
Cell phone and SMR	806-902	55
Unlicensed	902-928	10
Aero Radar, Military	1300-1400	3
Mobile Sat., GPS, Meteorologicial	1300-1400	3
Surveillance Radar	2686-2900	5

*M. A. McHenry, D. McCloske, D. Roberson, and J. T. MacDonald. Spectrum occupancy measurements in Chicago, Illinois. Technical report, Shared Spectrum Company and IIT Wireless Interference Lab Illinois Institute of Technology, Nov. 2005.

Importance and Methods of Cognitive Radio

Cognitive radio objectives:

- efficiently utilize the radio spectrum
- co-exist with primary users
- doesn't interfere with them (if possible), or interfere under tolerable limits

Spectrum usage methods:

- *interweave* cognitive networks
- *overlay* cognitive networks
- *underlay* (spectrum sharing) cognitive networks

Challenges in Spectrum Sensing

Main challenge in CR is spectrum sensing, due to;

- Uncertainties due to channel randomness
- Hidden PU problem
- Sensing duration and frequency
- Decision fusion in cooperative sensing
- Security

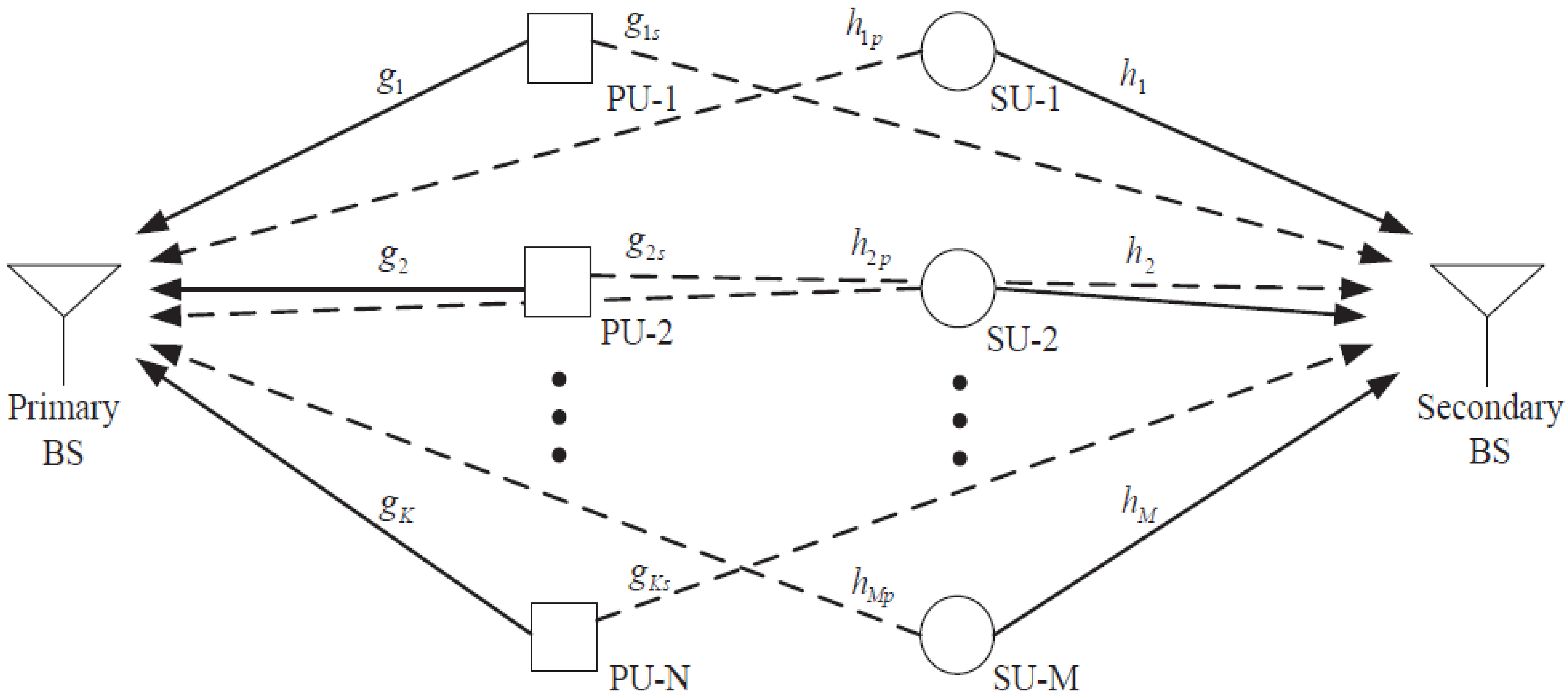
Unreliable information

Hence, the accuracy and reliability of the spectrum sensing information can be inherently suspicious and questionable.

Motivation

- Questions:
  - Why relying on spectrum sensing information?
  - What happens when we don’t have this information?
  - What is the basic scenario performance with limited information?
- Assumptions:
  - No spectrum sensing information available at the SUs
  - No information exchange overhead
  - No cooperation (delay)
  - Limited channel side information
  - SUs randomly (blindly) access subcarriers in the primary network
  - A stochastic model to capture the subcarrier collisions

System Model



Channel coeffs.: h_m, h_{mp}, g_n and g_{ns}
PU-n: nth primary user, SU-m: mth secondar user and BS: base station
(- -): interference-link (channel), (–): desired-link (channel)

Importance of Random Allocation

- A valid candidate for performance comparison benchmark in OFDM-based CR spectrum sharing systems with the availability of spectrum sensing information.
- The main benefit of random subcarrier utilization is to uniformly distribute the amount of SU's interference among the PUs' subcarriers.
- Lower complexity

Contributions of Random Allocation

- Statistics of number of subcarrier collisions
- SU's capacity expressions (average and inst.) for general fading
- Bounds and scaling laws of SU's capacity
- PDF and CDF of SU's capacity over Rayleigh fading channels
    - Utilize moment matching method and Moschopoulos PDF (1985)
- Multiuser diversity gain (MDG) analysis using extreme value theory
- Sequential subcarrier scheduling algorithm by exploiting MDG
- Analyzing the algorithm, and statistics of collisions in the algorithm

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S. Ekin, M. Abdallah, K. A. Qaraqe, and E. Serpedin, "Random Subcarrier Allocation in OFDM-Based Cognitive Radio Networks," *IEEE Transactions on Signal Processing*, vol. 60, no. 9, pp. 4758–4774, Sept. 2012.

PMF of Number of Subcarrier Collisions

Proposition

When the mth SU randomly utilizes F_m^S subcarriers from the set of F available subcarriers without replacement, and F_n^P subcarriers are being used by the nth PU, then the probability mass function (PMF) of number of subcarrier collisions, k_{nm}, follows the hypergeometric distribution, $k_{nm} \sim \texttt{HYPG}(F_m^S, F_n^P, F)$, and is expressed as:

$$Pr(K_{nm} = k_{nm}) = p(k_{nm}) = \binom{F_n^P}{k_{nm}} \binom{F - F_n^P}{F_m^S - k_{nm}} \Big/ \binom{F}{F_m^S},$$

The average number of subcarrier collisions is

$$\mathbb{E}[k_{nm}] = \frac{F_m^S F_n^P}{F},$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator.

PMF of Number of Subcarrier Collisions

Proposition (Cont.)

Let $\mathbf{k}_m = [k_{1m}, k_{2m}, \ldots, k_{Nm}, k_{fm}]^T \in \mathbb{Z}_{0+}^{N+1}$ *represent the number of collisions of the* m*th SU with* N *PUs and the collision-free subcarriers,* k_{fm}*. Then, the PMF of* $\mathbf{k}_m$ *is given by*

$$Pr(\mathbf{K}_m = \mathbf{k}_m) = \left[\binom{F_f}{k_{fm}} \prod_{n=1}^{N} \binom{F_n^P}{k_{nm}} \right] \Big/ \binom{F}{F_m^S},$$

That is the multivariate hypergeometric distribution as $\mathbf{k}_m \sim \mathit{M\text{-}HYPG}(F_m^S, \mathbf{F}^{\mathbf{P}}, F)$ *with* $\mathbf{F}^{\mathbf{P}} = [F_1^P, F_2^P, \ldots, F_N^P, F_f]^T \in \mathbb{Z}_{0+}^{N+1}$ *and the support of* $\mathbf{k}_m$ *as*

$$\left\{ \mathbf{k}_m : \sum_{n=1}^{N} k_{nm} + k_{fm} = F_m^S \text{ and } k_{nm} \in \left[\left(F_m^S + F_n^P - F \right)^+, \ldots, \min\{F_m^S, F_n^P\} \right] \right\},$$

where $F_f = F - \sum_{n=1}^{N} F_n^P$ *is the number of free subcarriers and* $(x)^+ = \max\{0, x\}$*.*

SU Instantaneous Capacity

Total Sum Capacity with Subcarrier Collisions

Let $S_{m,i}^{I,n}$ and $S_{m,i}^{NI}$ be the signal-to-interference plus noise ratio (SINR) for ith subcarrier of the mth SU with “interference" and “no-interference" from the nth PU, respectively. The *total sum capacity* of SU can be defined as:

$$C_m^1 = \sum_{i=1}^{k_{nm}} \underbrace{\log\left(1 + S_{m,i}^{I,n}\right)}_{C_{m,i}^{I,n}} + \sum_{i=1}^{k_{fm}} \underbrace{\log\left(1 + S_{m,i}^{NI}\right)}_{C_{m,i}^{NI}} .$$

Total Sum Capacity with Subcarrier Collisions - Multiple PUs

$$C_m = \underbrace{\sum_{n=1}^{N} \overbrace{\sum_{i=1}^{k_{nm}} C_{m,i}^{I,n}}^{C_m^{I,n}}}_{C_m^I} + \underbrace{\sum_{i=1}^{k_{fm}} C_{m,i}^{NI}}_{C_m^{NI}} .$$

SU Average Capacity over General Fading

Theorem

The average capacity of mth SU in the presence of a single PU is given as

$$\mathbb{E}[C_m^1] = \frac{F_m^S}{F}\left[F_n^P\left(\mathbb{E}[C_{m,i}^{I,n}] - \mathbb{E}[C_{m,i}^{NI}]\right) + F\mathbb{E}[C_{m,i}^{NI}]\right],$$

$C_{m,i}^{I,n}$ and $C_{m,i}^{NI}$ represent the ith subcarrier capacity of mth SU with "interference" and "no-interference" from the nth PU, respectively.

Corollary

The average capacity of mth SU in the presence of multiple N PUs is given by

$$\mathbb{E}[C_m] = \frac{F_m^S}{F}\left[\sum_{n=1}^{N} F_n^P \mathbb{E}[C_{m,i}^{I,n}] + F_f \mathbb{E}[C_{m,i}^{NI}]\right].$$

Bounds & Scaling Laws

Corollary

The upper and lower bounds on the average capacity of SU in the presence of a single PU are given by:

$$k_{nm}^{\max}\mathbb{E}[C_{m,i}^{I,n}] + k_{fm}^{\min}\mathbb{E}[C_{m,i}^{NI}] \leq \mathbb{E}[C_m^1] \leq k_{nm}^{\min}\mathbb{E}[C_{m,i}^{I,n}] + k_{fm}^{\max}\mathbb{E}[C_{m,i}^{NI}],$$

where $k_{nm}^{\min} = \left(F_m^S + F_n^P - F\right)^+$, $k_{nm}^{\max} = \min\left\{F_m^S, F_n^P\right\}$, $k_{fm}^{\max} = F_m^S - k_{nm}^{\min}$, *and* $k_{fm}^{\min} = F_m^S - k_{nm}^{\max}$.

Corollary

The average capacity of the m*th secondary user in the presence of a single PU scales with number of subcarriers* F, F_m^S *and* F_n^P *as* $\Theta\left(1 + \frac{1}{F}\right)$, $\Theta\left(F_m^S\right)$ *and* $\Theta\left(1 - F_n^P\right)$, *respectively.*

Received SINR of ith Subcarrier

- The transmit power of the mth SU corresponding to the ith subcarrier is given by

$$P_{m,i}^{T} = \begin{cases} P_{m,i}\ , & \Psi_i \geq P_{m,i} h_{mp,i} \\ \frac{\Psi_i}{h_{mp,i}}, & \Psi_i < P_{m,i} h_{mp,i} \end{cases} = \min\left\{P_{m,i}, \frac{\Psi_i}{h_{mp,i}}\right\}, \quad i = 1, \ldots, F.$$

  Ψ_i: Interference temperature constraint.

- Then, the received SINR of the mth SU's ith subcarrier at the SBS is

$$S_{m,i}^{I,n} = \frac{h_{m,i} P_{m,i}^{T}}{I_{n,i}^{P} + \eta}, \quad \text{for } n = 1, \ldots, N,$$

  $I_{n,i}^{P} = P_{n,i} g_{ns,i}$ stands for the mutual interference caused by nth PU and η is the AWGN noise variance.

- In collision-free case, the received SINR is

$$S_{m,i}^{NI} = \frac{h_{m,i} P_{m,i}^{T}}{\eta}.$$

Summary of Derivations

- Derived the PDFs and CDFs of $S_{m,i}^{NI}$ and $S_{m,i}^{I,n}$
- The PDFs and CDFs of $C_{m,i}^{I,n}$ and $C_{m,i}^{NI}$ are obtained by transformation of RVs.
- Intractable to obtain explicit closed-form of the SU capacity with CF and MGF approaches.
- Using moment matching method to approximate the PDFs with Gamma distribution
- Recall the SU capacity:

$$C_m = \underbrace{\sum_{i=1}^{k_{1m}} \underbrace{\log\left(1+S_{m,i}^{I,1}\right)}_{C_{m,i}^{I,1}}}_{C_m^{I,1}} + \cdots + \underbrace{\sum_{i=1}^{k_{Nm}} \underbrace{\log\left(1+S_{m,i}^{I,N}\right)}_{C_{m,i}^{I,N}}}_{C_m^{I,N}} + \underbrace{\sum_{i=1}^{k_{fm}} \underbrace{\log\left(1+S_{m,i}^{NI}\right)}_{C_{m,i}^{NI}}}_{C_m^{NI}}$$

- Utilized Moschopoulos PDF for independent but not necessarily identically distributed Gamma variates to obtain the PDF and CDF of SU's capacity expressions.

PDF of SU Capacity

$$f_{C_m}(x) = \sum_{k_{1m}} \sum_{k_{2m}} \cdots \sum_{k_{Nm}} \sum_{k_{fm}} \left\{ \left[\binom{F_f}{k_{fm}} \Big/ \binom{F}{F_m^S} \right] \right.$$

$$\times \prod_{n=1}^{N} \binom{F_n^P}{k_{nm}} \left(\frac{\beta_{\min}}{\beta^{NI}} \right)^{\alpha^{NI} k_{fm}} \prod_{n=1}^{N} \left(\frac{\beta_{\min}}{\beta_n^I} \right)^{\alpha_n^I k_{nm}}$$

$$\left. \times \sum_{k=0}^{\infty} \frac{\delta_k x^{\sum_{n=1}^{N} \alpha_n^I k_{nm} + \alpha^{NI} k_{fm} + k - 1} \exp\left(-\frac{x}{\beta_{\min}} \right)}{\beta_{\min}^{\sum_{n=1}^{N} \alpha_n^I k_{nm} + \alpha^{NI} k_{fm} + k} \Gamma\left(\sum_{n=1}^{N} \alpha_n^I k_{nm} + \alpha^{NI} k_{fm} + k \right)} U(x) \right\}.$$

$\beta_{\min} = \min\{\beta_1^I, \beta_2^I, \ldots, \beta_N^I, \beta^{NI}\}$, and the coefficients δ_k are obtained recursively as follows:

$$\delta_0 = 1$$

$$\delta_k = \frac{1}{k+1} \sum_{i=1}^{k+1} \left[\sum_{j=1}^{N} \alpha_i^I k_{jm} \left(1 - \frac{\beta_{\min}}{\beta_j^I} \right)^i + \alpha^{NI} k_{fm} \left(1 - \frac{\beta_{\min}}{\beta^{NI}} \right)^i \right] \delta_{k+1-i}$$

for $k = 0, 1, 2, \ldots$

Outage Probability of SU Capacity

- The outage probability is a common performance metric in fading environments:

$$P^{\text{out}}_{C_m}(\varphi_{\text{th}}) = Pr\left(C_m < \varphi_{\text{th}}\right) = \int_0^{\varphi_{\text{th}}} f_{C_m}(x)\mathrm{d}x,$$

which is the cumulative distribution function (CDF) of the SU capacity over the outage threshold φ_{th} [dB].

- The CDF of C_m can be expressed as

$$F_{C_m}(x) = \sum_{k_{1m}} \sum_{k_{2m}} \cdots \sum_{k_{Nm}} \sum_{k_{fm}} \left\{ \left[\binom{F_f}{k_{fm}} \Big/ \binom{F}{F^S_m} \right] \prod_{n=1}^{N} \binom{F^P_n}{k_{nm}} \left(\frac{\beta_{\min}}{\beta^{NI}}\right)^{\alpha^{NI} k_{fm}} \times \prod_{n=1}^{N} \left(\frac{\beta_{\min}}{\beta^I_n}\right)^{\alpha^I_n k_{nm}} \sum_{k=0}^{\infty} \delta_k \mathcal{P}\left(\sum_{n=1}^{N} \alpha^I_n k_{nm} + \alpha^{NI} k_{fm} + k, \frac{x}{\beta_{\min}} \right) \right\}.$$

*$\mathcal{P}(a,z) = \frac{\gamma(a,z)}{\Gamma(a)} = 1 - \frac{\Gamma(a,z)}{\Gamma(a)}$: normalized incomplete Gamma function.

Opportunistic Scheduling

- The SU, which provides the best instantaneous capacity, is selected as

$$C_{\max} = \max_{m \in [1,M]} C_m$$

- For fairness, assume that each SU's data rate is the same.
- Using order statistics, the PDF of $C_{\max}$ is expressed as

$$f_{C_{\max}}(x) = M f_{C_m}(x) F_{C_m}(x)^{M-1}.$$

- The average of $C_{\max}$ is:

$$\mathbb{E}[C_{\max}] = \int_{-\infty}^{\infty} x \, f_{C_{\max}}(x) \mathrm{d}x \Rightarrow \text{ hard to obtain!}$$

- Asymptotically analyze by using extreme value theory.

Asymptotic Analysis

- The CDF of $C_{\max}$ belongs to domain of attraction of Gumbel-type with limiting CDF as

$$\hat{F}_{C_{\max}}(x) = \exp\left(-\exp\left(-\frac{x-b_M}{a_M}\right)\right).$$

- The limiting PDF of $C_{\max}$ is

$$\hat{f}_{C_{\max}}(x) = \frac{1}{a_M}\exp\left(-\frac{x-b_M}{a_M}\right)\exp\left(-\exp\left(-\frac{x-b_M}{a_M}\right)\right).$$

Theorem

As the number of SUs M goes to infinity, the average capacity of $C_{\max}$ converges to

$$\mathbb{E}[C_{\max}] = b_M + E_1 a_M,$$

where $E_1 = 0.5772\ldots$ is Euler's constant, $a_M = [M f_{C_m}(b_M)]^{-1}$ and $b_M = F_{C_m}^{-1}(1-\frac{1}{M})$.

ALGORITHM: Random Subcarrier Allocation

1. *Initialization*
   - Assume $F_m^S = F^S \; \forall m \in [1, M]$ and a single PU is available, $n = 1$.
   - Set the number of available subcarriers to F and index $t = 1$.
2. *Subcarrier assignment step*
   - Randomly sample a set of subcarriers, F_t^R, with cardinality of F^S from set F: $k_{nm} \sim \mathtt{HYPG}(F^S,\; F_n^P,\; F)$.
   - Assign the set F_t^R to all $M - t + 1$ SUs.
3. *Capacity calculation step*
   - For $m = 1, \ldots, M - t + 1$, SUs evaluate their capacities with the given random set of subcarriers: $C_m|F_t^R$.
   - SUs send *feedback* for the calculated capacities to the *central control entity* (SBS or CR Network Manager).
4. *Selection step*
   - Choose the SU that provides the best capacity:
     If $t = 1$, $m_t^* = \underset{m \in [1,M]}{\arg\max} \left(C_m | F_t^R \right)$ else $m_t^* = \underset{m \in [1,M] \setminus \left[m_1^*, m_{t-1}^* \right]}{\arg\max} \left(C_m | F_t^R \right)$.
5. *Updating the subcarrier sets step*
   - Remove the sampled (total of collided and collision-free) subcarriers from the available set of subcarriers: $F \leftarrow F - F_t^R$.
   - Set $t \leftarrow t + 1$ and go to Step 2 until $t = \hat{M}$.
6. *Sum capacity evaluation step*: $C_{\text{sum}} = \sum_{t=1}^{\hat{M}} C_{m_t^*}$.

Sum Capacity

Theorem

The sum capacity of $\hat{M}$ selected SUs in the sequential scheduling algorithm for $M \gg \hat{M}$ is approximated[a] by

$$\mathbb{E}[C_{\text{sum}}] \approx \hat{M}\mathbb{E}[C_{m_1^*}],$$

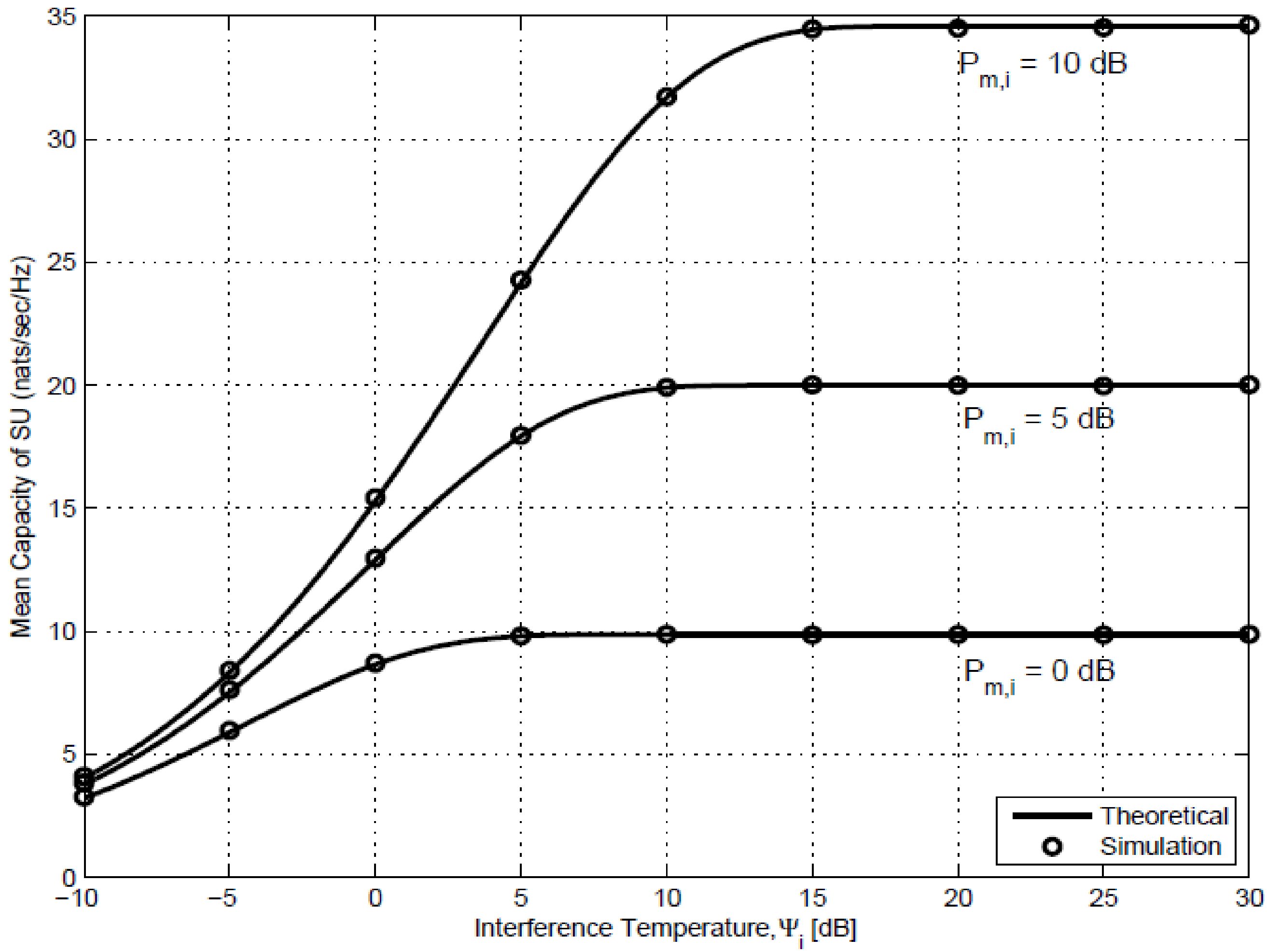
and as $M \to \infty$, it converges to

$$\mathbb{E}[C_{\text{sum}}] = \hat{M}[b_M + E_1 a_M],$$

where m_1^* is the index of the first selected best SU and defined as $m_1^* = \underset{m \in [1,M]}{\arg\max}\, C_m$.

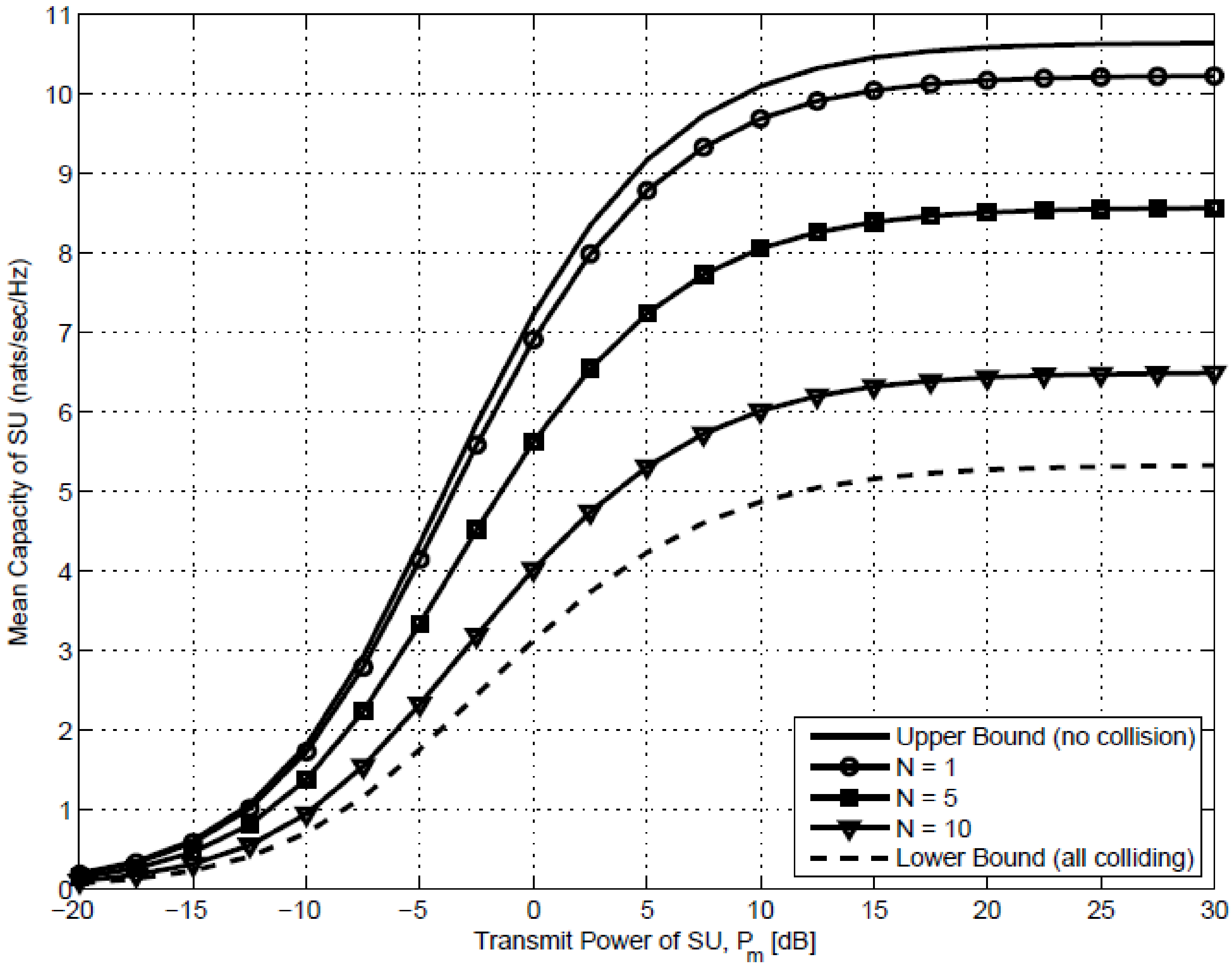
[a] Since $\mathbb{E}[C_{m_1^*}] \geq \mathbb{E}[C_{m_j^*}], \forall j \in [1, \hat{M}]$, it can also be considered as a very tight upper bound for $M \gg \hat{M}$ as $\mathbb{E}[C_{\text{sum}}] \leq \hat{M}\mathbb{E}[C_{m_1^*}]$.

Average Capacity vs Interference Temperature



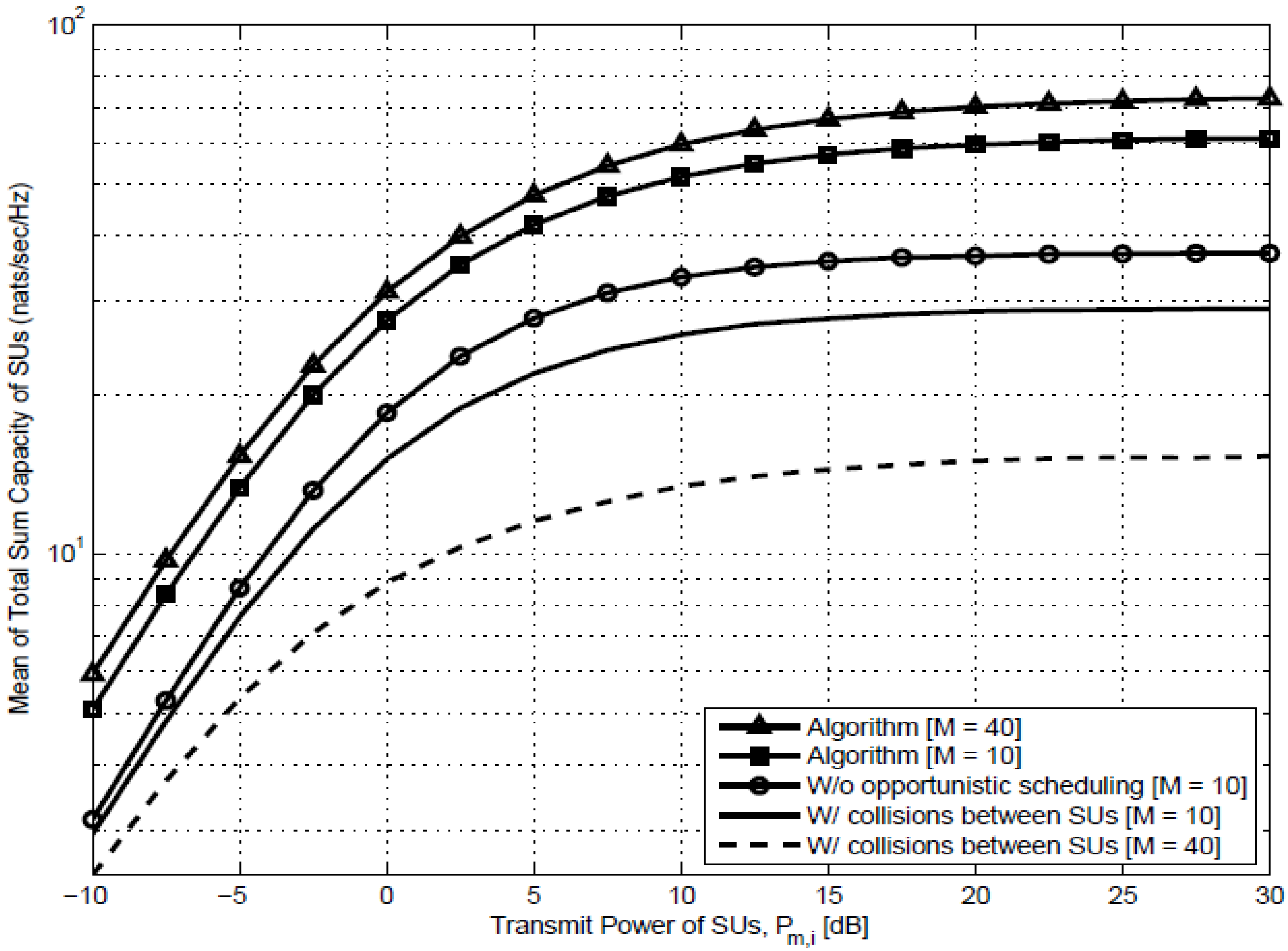
For $F_m^S = 20$, $F_n^P = 30$, $F = 128$ and $P_{n,i} = 10$ dB.

Multiple PUs



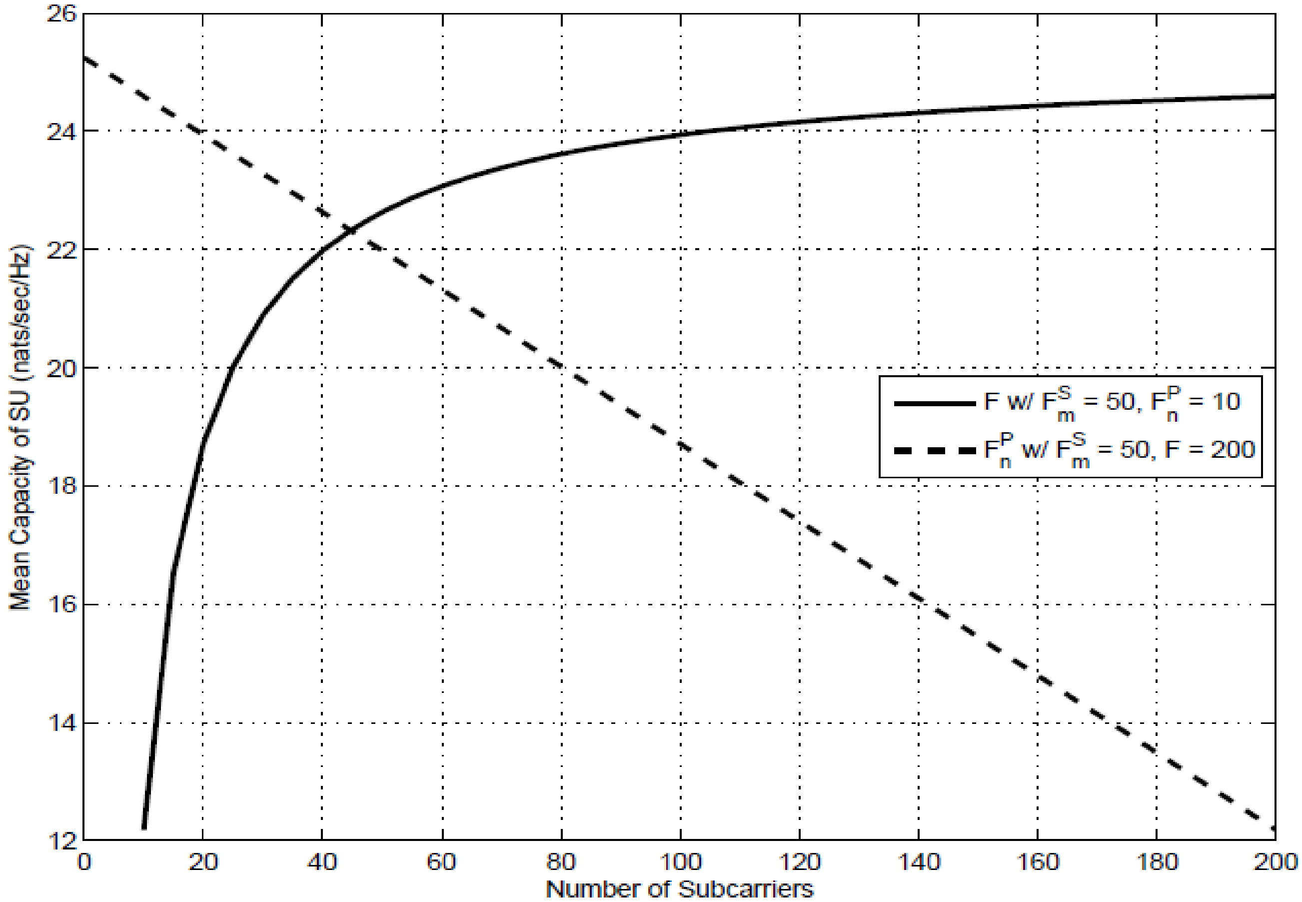
For $F_m^S = 20$, $F = 128$, $\Psi_i = -5$ dB, $F_n^P = 10$ and $P_{n,i} = 5$ dB, $n = 1, \ldots$

Scheduling Algorithm



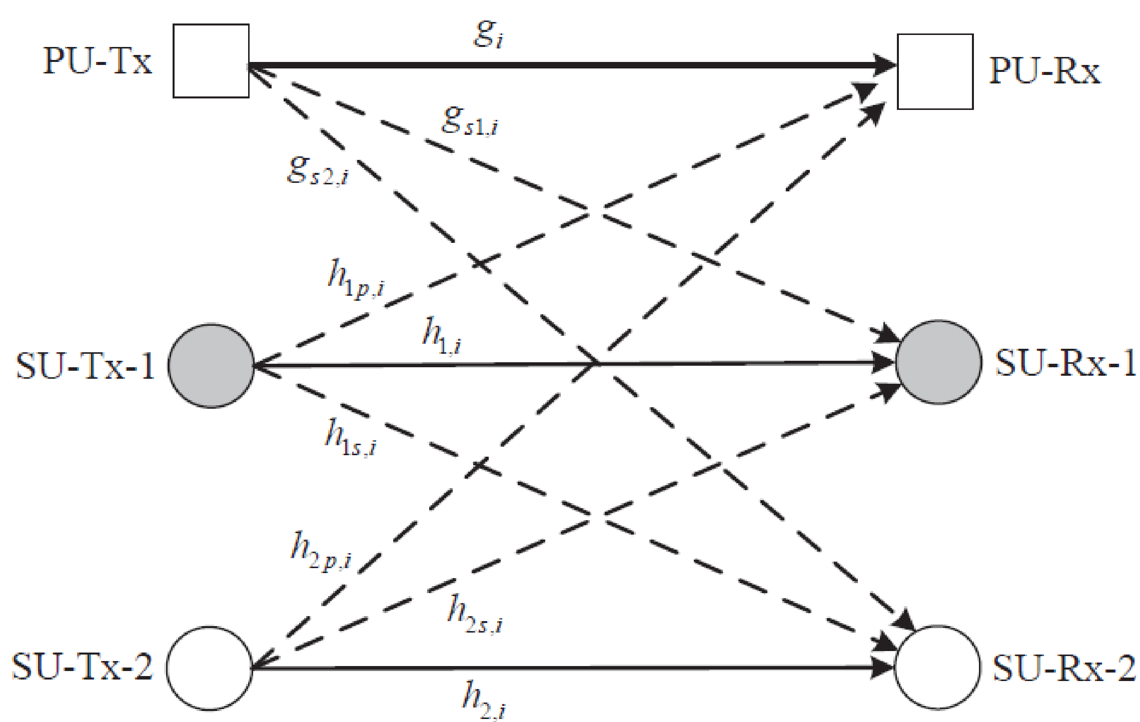
For $\hat{M} = 5$, $F_m^S = 10$, $m = 1, \ldots, M$, $F_n^P = 40$, $F = 100$, $P_{n,i} = 10$ dB and $\Psi_i = 0$ dB.

Scaling Laws of SU Capacity



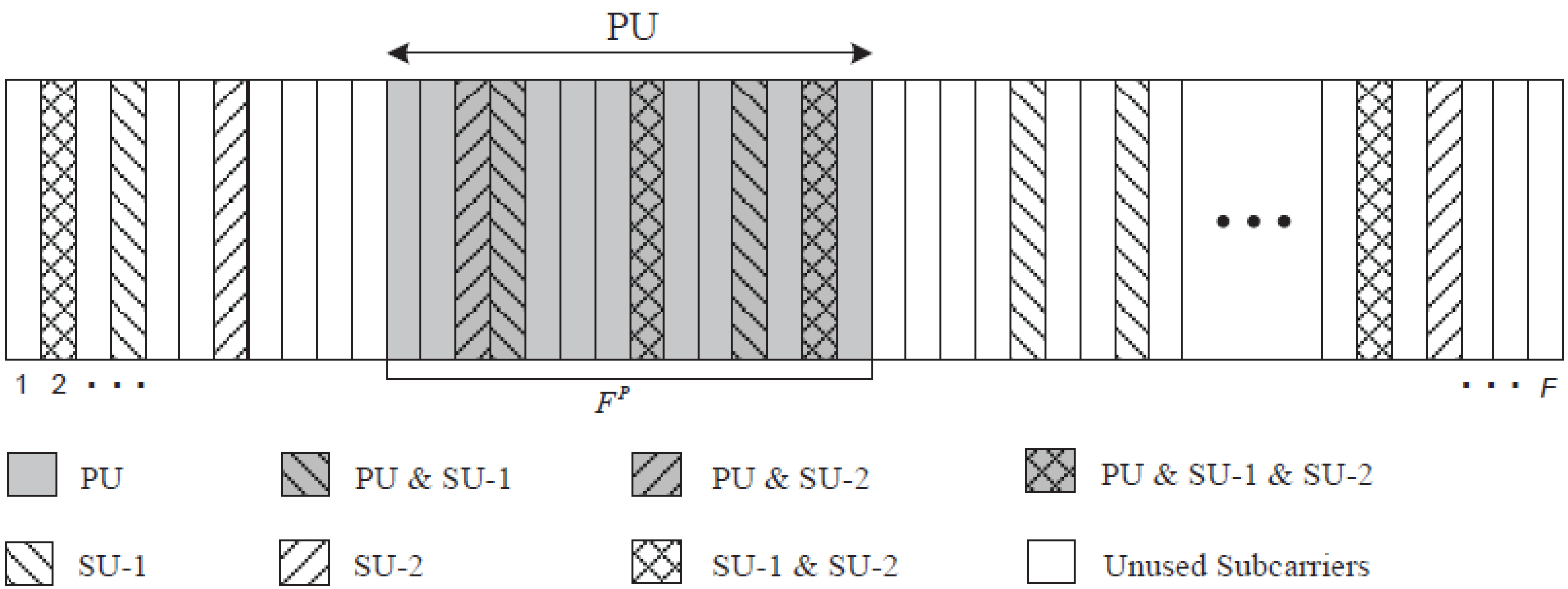
For $P_{m,i} = 5$ dB, $P_{n,i} = 10$ dB and $\Psi_i = -5$ dB.

Subcarrier Channel Model



*The performance of shaded SU pairs (SU-1) is of interest.

OFDM Subcarrier Collision Model



1. Collisions between SU-1, PU and SU-2 subcarriers: k_{p12}
2. Collisions *only* between SU-1 and PU subcarriers: k^o_{p1}
3. Collisions *only* between SU-1 and SU-2 subcarriers: k^o_{12}
4. Collisions-free subcarriers of SU-1: k_{f1}

Conclusions and Future Directions

- Outline of Contributions:
  - Stochastic model to capture subcarrier collisions in OFDM-based CR spectrum sharing systems
  - Inter-cell subcarrier collisions with random subcarrier requirements
  - Hyper fading channel model for fits dynamic nature of CR environments
- Future Research Directions:
  - Comparison of the proposed scheme when spectrum sensing is available
  - Deriving the collision model when there are inter-cell collisions between multiple SUs belonging different cells
  - Investigating the importance of uniform interference distribution due to random access in different scenarios
  - Analyze the impacts of random access on PUs performance without transmit power adaptation