Turbo Codes in UMTS/ WiMAX/ LTE Systems: Solutions for an Efficient FPGA Implementation

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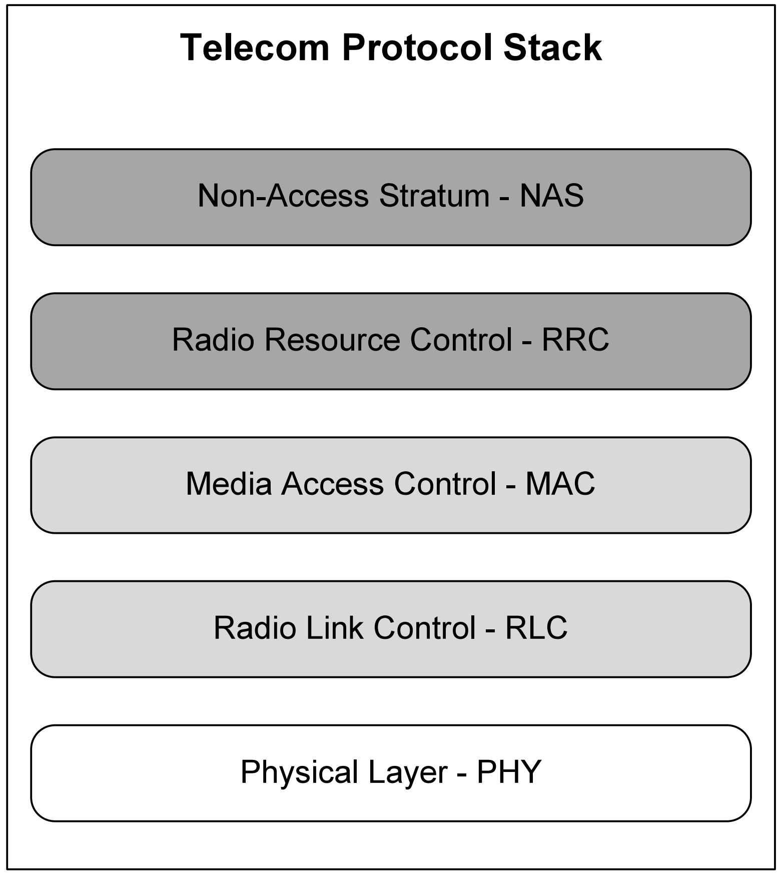
INTRODUCTION



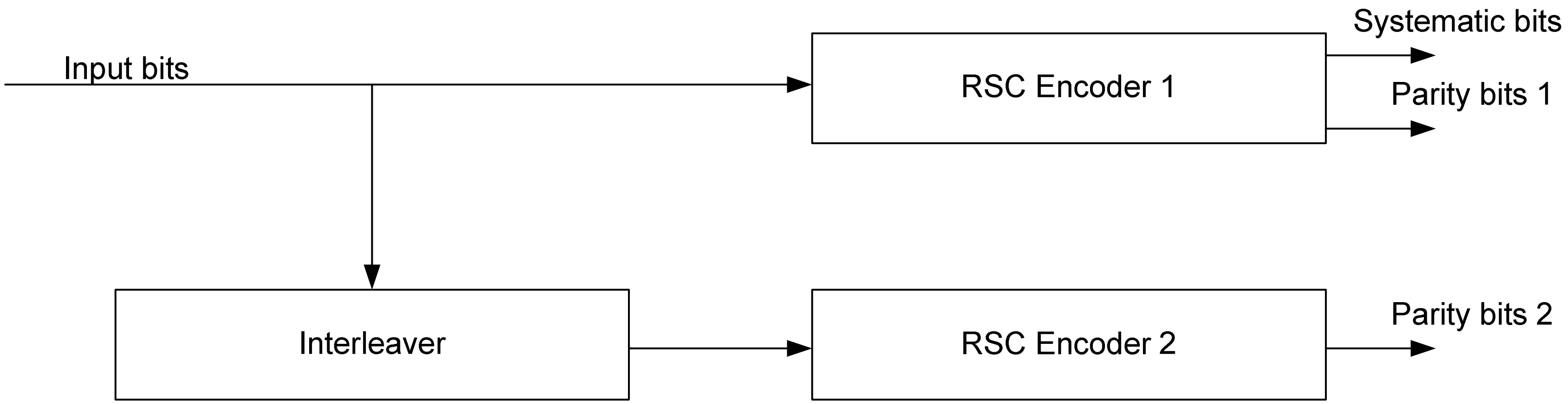
INTRODUCTION

- Forward Error Coding – FEC
  - Source coding
  - Channel coding
    - Block codes (Hamming, Hadamard, cyclic)
    - Convolutional codes
      - Non-systematic convolutional codes NSC
      - Recursive systematic covolutional codes RSC
    - Turbo codes
      - Systematic codes used in a pseudo-random manner

- All codes are good, except the ones we can think of (Jacob Wolfowitz)

Turbo Codes

- Turbo coding principle

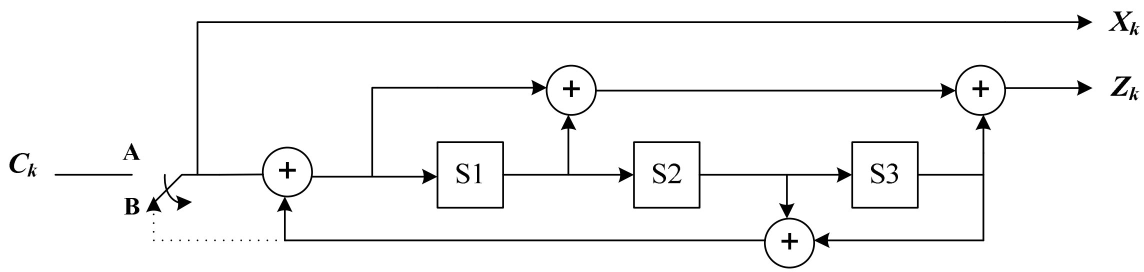


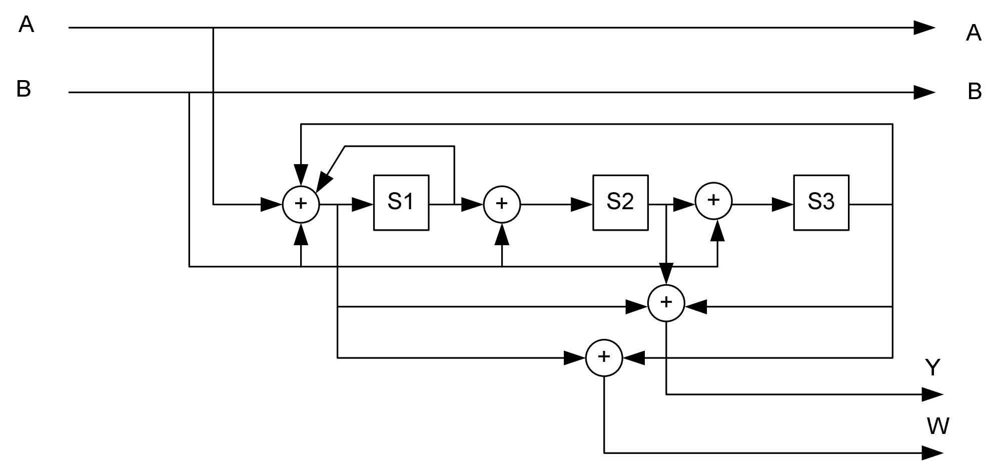
- Coding rate $R_c = \frac{1}{3}$

- [1] C. Berrou, A. Glavieux, and P. Thitimajshima, Near Shannon limit error-correcting coding and decoding: Turbo Codes, *IEEE Proceedings of the Int. Conf. on Communications*, Geneva, Switzerland, May 1993, pp. 1064-1070.
- [2] C. Berrou and A. Glavieux, Near optimum error correcting coding and decoding: Turbo-Codes, *IEEE Trans. Communications*, vol. 44, no. 10, pp. 1261-1271, Oct. 1996.
- [3] C. Berrou and M. Jézéquel, Non binary convolutional codes for turbo coding, *Electronics Letters*, vol. 35, no. 1, pp. 9-40, Jan. 1999.

TURBO CODES

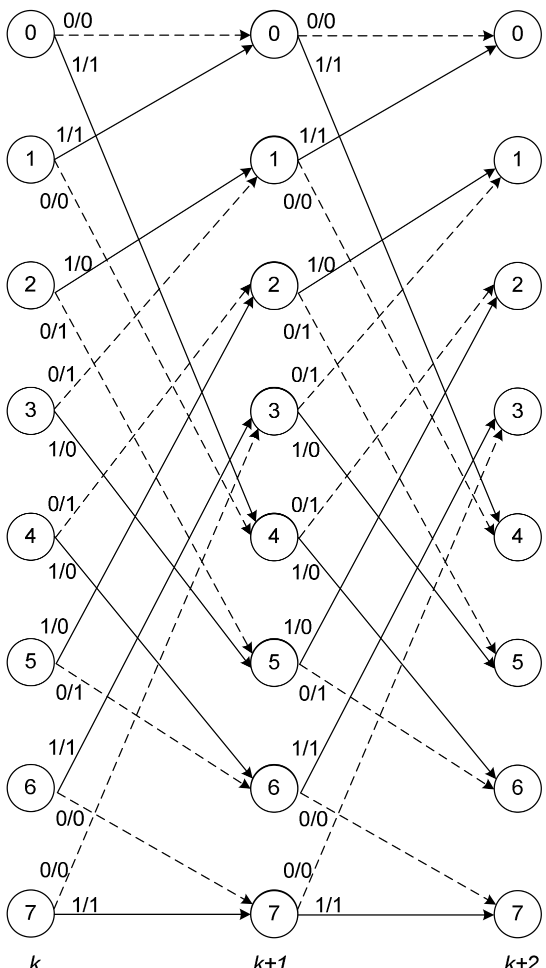
- UMTS/ WiMAX/ LTE RSC turbo encoder

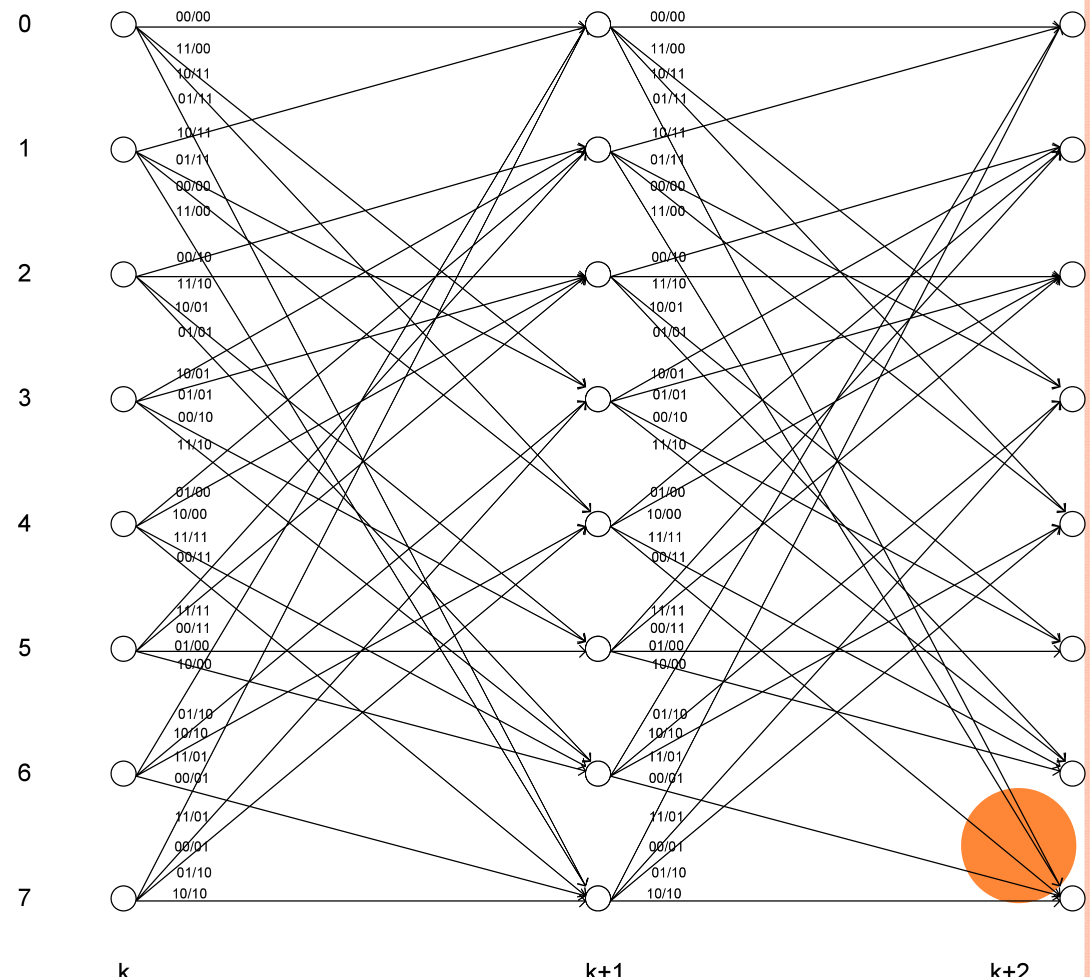




TURBO CODES

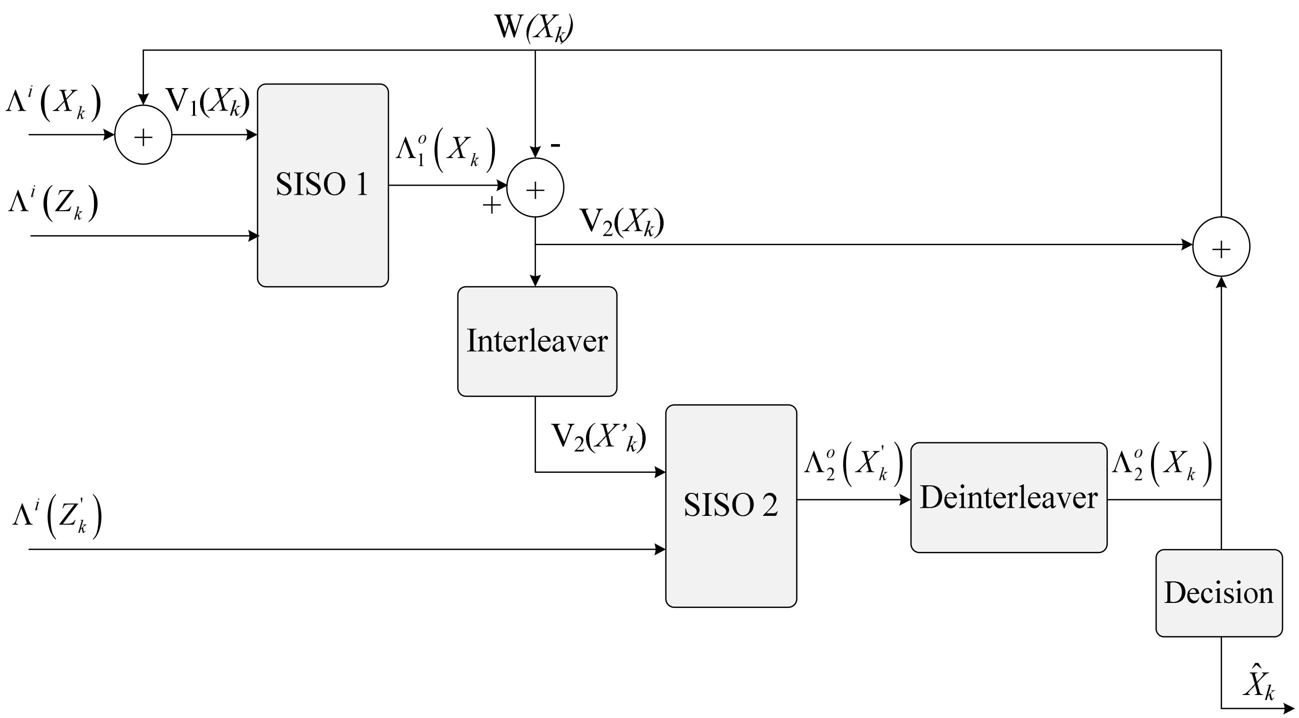
- UMTS/ WiMAX/ LTE RSC trellis





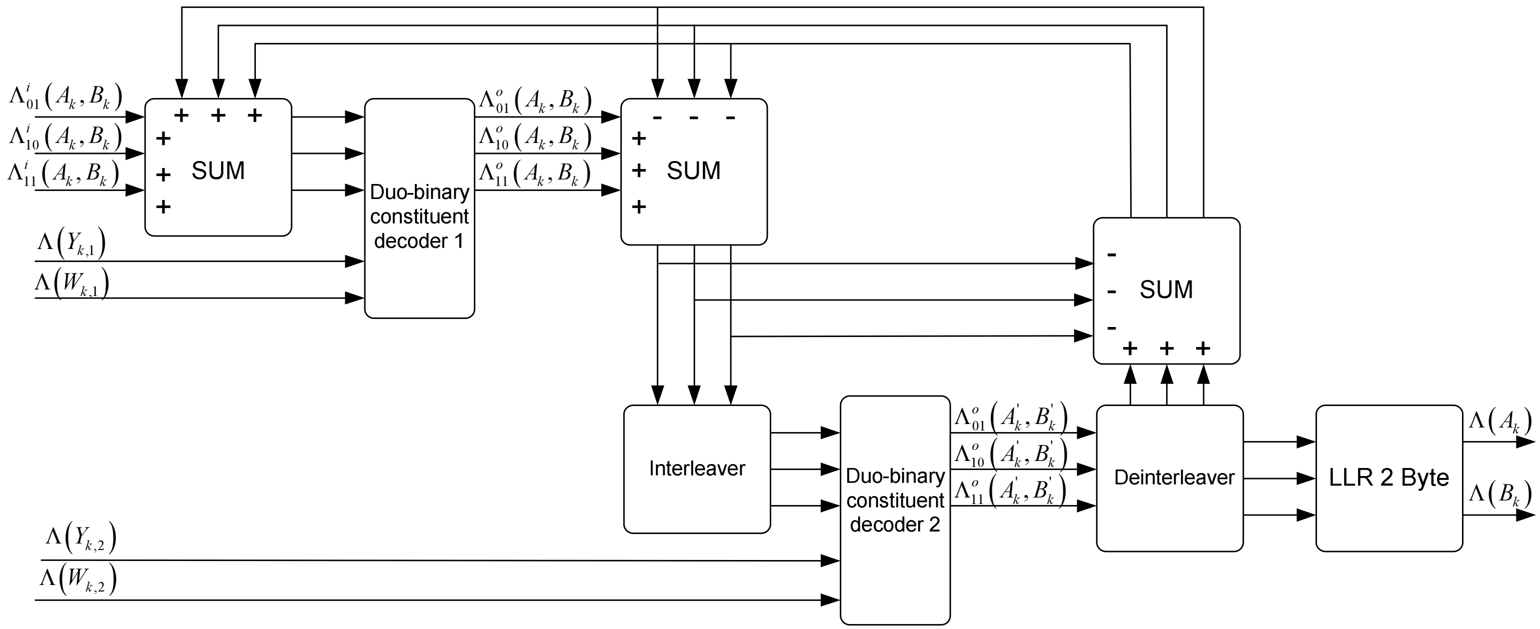
TURBO DECODER

- UMTS/ LTE RSC turbo decoder



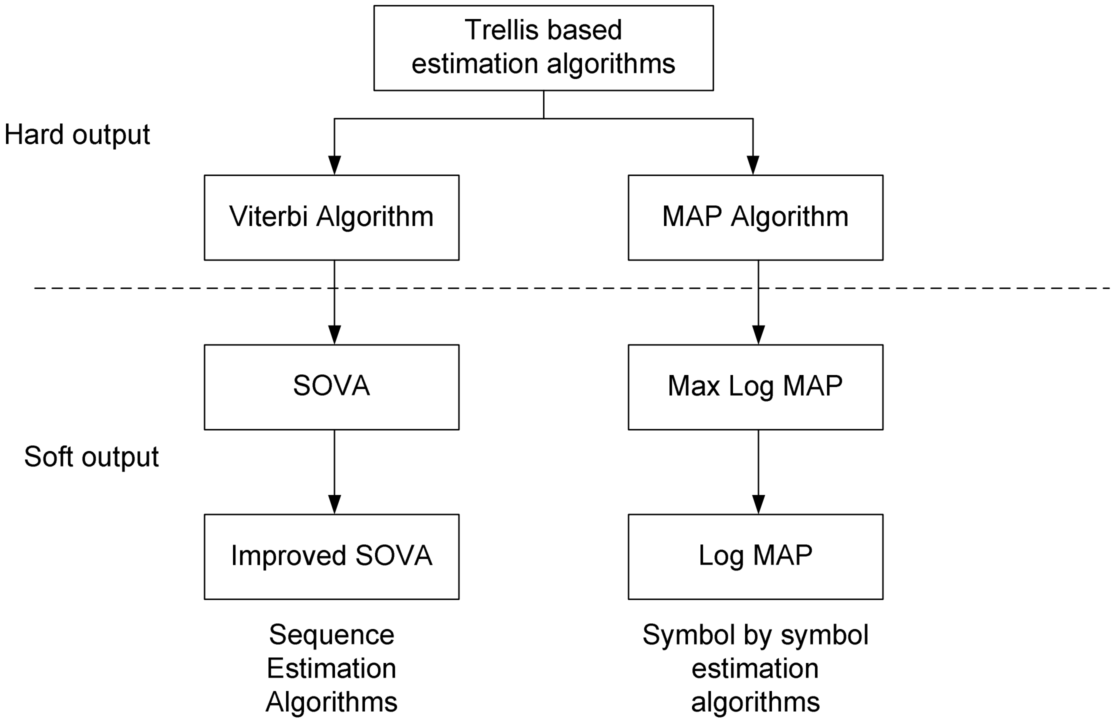
TURBO DECODER

- WiMAX RSC turbo decoder



Turbo Decoder

- Decoding algorithm



$$\max^*(x,y) = \ln(e^x + e^y) =$$
$$\max(x,y) + \ln(1 + e^{-|y-x|}) \approx \max(x,y).$$

Turbo Decoder

- Max-Log MAP – WiMAX

$$\Lambda_{a,b}(A_k, B_k) = \log \frac{P(A_k = a, B_k = b)}{P(A_k = 0, B_k = 0)}$$

$$\overline{\gamma}_k\left(S_k^i \to S_{k+1}^j\right) = \Lambda_{a,b}^{i}(A_k, B_k) + w\Lambda(W_k) + y\Lambda(Y_k)$$

$$\overline{\alpha}_{k+1}^{'}\left(S_{k+1}^j\right) = \max_{S_k^i \to S_{k+1}^j}\left\{\overline{\alpha}_k\left(S_k^i\right) + \overline{\gamma}_{k+1}\left(S_k^i \to S_{k+1}^j\right)\right\}$$

$$\overline{\alpha}_{k+1}\left(S_{k+1}^j\right) = \overline{\alpha}_{k+1}^{'}\left(S_{k+1}^j\right) - \overline{\alpha}_{k+1}^{'}\left(S_{k+1}^0\right)$$

$$\overline{\beta}_k^{'}\left(S_k^i\right) = \max_{S_k^i \to S_{k+1}^j}\left\{\overline{\beta}_{k+1}\left(S_{k+1}^j\right) + \overline{\gamma}_{k+1}\left(S_k^i \to S_{k+1}^j\right)\right\}$$

$$Z_k\left(S_k^i \to S_{k+1}^j\right) = \overline{\alpha}_k\left(S_k^i\right) + \overline{\gamma}_{k+1}\left(S_k^i \to S_{k+1}^j\right) + \overline{\beta}_{k+1}\left(S_{k+1}^j\right)$$

$$t_k(a,b) = \max_{S_k^i \to S_{k+1}^j:(a,b)}\left\{Z_k\right\}$$

$$\Lambda_{a,b}^{o}(A_k, B_k) = t_k(a,b) - t_k(0,0)$$

$$\Lambda(A_k) = \max\left\{\Lambda_{1,0}^{o}(A_k, B_k), \Lambda_{1,1}^{o}(A_k, B_k)\right\} - \max\left\{\Lambda_{0,0}^{o}(A_k, B_k), \Lambda_{0,1}^{o}(A_k, B_k)\right\}$$

$$\Lambda(B_k) = \max\left\{\Lambda_{0,1}^{o}(A_k, B_k), \Lambda_{1,1}^{o}(A_k, B_k)\right\} - \max\left\{\Lambda_{0,0}^{o}(A_k, B_k), \Lambda_{1,0}^{o}(A_k, B_k)\right\}$$

Turbo Decoder

- Max-Log MAP – WiMAX

$$\gamma_k\left(S_k^0 \to S_{k+1}^0\right) = 0$$
$$\gamma_k\left(S_k^0 \to S_{k+1}^3\right) = \Lambda_{11}^0\left(A_k, B_k\right)$$
$$\gamma_k\left(S_k^0 \to S_{k+1}^4\right) = \Lambda_{10}^0\left(A_k, B_k\right) + \Lambda(Y_k) + \Lambda(W_k)$$
$$\gamma_k\left(S_k^0 \to S_{k+1}^7\right) = \Lambda_{01}^0\left(A_k, B_k\right) + \Lambda(Y_k) + \Lambda(W_k)$$

$$\gamma_k\left(S_k^1 \to S_{k+1}^0\right) = \Lambda_{10}^0\left(A_k, B_k\right) + \Lambda(Y_k) + \Lambda(W_k)$$
$$\gamma_k\left(S_k^1 \to S_{k+1}^3\right) = \Lambda_{01}^0\left(A_k, B_k\right) + \Lambda(Y_k) + \Lambda(W_k)$$
$$\gamma_k\left(S_k^1 \to S_{k+1}^4\right) = 0$$
$$\gamma_k\left(S_k^1 \to S_{k+1}^7\right) = \Lambda_{11}^0\left(A_k, B_k\right)$$

$$\overline{\beta}_k^{a'}\left(S_k^0\right) = \overline{\beta}_{k+1}\left(S_{k+1}^0\right) + \overline{\gamma}_{k+1}\left(S_k^0 \to S_{k+1}^0\right)$$
$$\overline{\beta}_k^{b'}\left(S_k^0\right) = \overline{\beta}_{k+1}\left(S_{k+1}^3\right) + \overline{\gamma}_{k+1}\left(S_k^0 \to S_{k+1}^3\right)$$
$$\overline{\beta}_k^{c'}\left(S_k^0\right) = \overline{\beta}_{k+1}\left(S_{k+1}^4\right) + \overline{\gamma}_{k+1}\left(S_k^0 \to S_{k+1}^4\right)$$
$$\overline{\beta}_k^{d'}\left(S_k^0\right) = \overline{\beta}_{k+1}\left(S_{k+1}^7\right) + \overline{\gamma}_{k+1}\left(S_k^0 \to S_{k+1}^7\right)$$

$$\overline{\alpha}_{k+1}^{a'}\left(S_k^0\right) = \overline{\alpha}_k\left(S_k^0\right) + \overline{\gamma}_{k+1}\left(S_k^0 \to S_{k+1}^0\right)$$
$$\overline{\alpha}_{k+1}^{b'}\left(S_k^0\right) = \overline{\alpha}_k\left(S_k^1\right) + \overline{\gamma}_{k+1}\left(S_k^1 \to S_{k+1}^0\right)$$
$$\overline{\alpha}_{k+1}^{c'}\left(S_k^0\right) = \overline{\alpha}_k\left(S_k^6\right) + \overline{\gamma}_{k+1}\left(S_k^6 \to S_{k+1}^0\right)$$
$$\overline{\alpha}_{k+1}^{d'}\left(S_k^0\right) = \overline{\alpha}_k\left(S_k^7\right) + \overline{\gamma}_{k+1}\left(S_k^7 \to S_{k+1}^0\right)$$

$$\overline{\beta}_k^{'}\left(S_k^0\right) = \max\left\{\overline{\beta}_k^{a'}\left(S_k^0\right), \overline{\beta}_k^{b'}\left(S_k^0\right), \overline{\beta}_k^{c'}\left(S_k^0\right), \overline{\beta}_k^{d'}\left(S_k^0\right)\right\}$$

$$\overline{\alpha}_{k+1}^{'}\left(S_{k+1}^0\right) = \max\left\{\overline{\alpha}_{k+1}^{a'}\left(S_{k+1}^0\right), \overline{\alpha}_{k+1}^{b'}\left(S_{k+1}^0\right), \overline{\alpha}_{k+1}^{c'}\left(S_{k+1}^0\right), \overline{\alpha}_{k+1}^{d'}\left(S_{k+1}^0\right)\right\}$$

TURBO DECODER

- Max-Log MAP – WiMAX

$$Z_k^a\left(S_k^0 \rightarrow S_{k+1}^0\right)=\bar{\alpha}_k\left(S_k^0\right)+\bar{\gamma}_{k+1}\left(S_k^0 \rightarrow S_{k+1}^0\right)+\bar{\beta}_{k+1}\left(S_{k+1}^0\right)$$
$$Z_k^b\left(S_k^1 \rightarrow S_{k+1}^4\right)=\bar{\alpha}_k\left(S_k^1\right)+\bar{\gamma}_{k+1}\left(S_k^1 \rightarrow S_{k+1}^4\right)+\bar{\beta}_{k+1}\left(S_{k+1}^4\right)$$
$$Z_k^c\left(S_k^2 \rightarrow S_{k+1}^1\right)=\bar{\alpha}_k\left(S_k^2\right)+\bar{\gamma}_{k+1}\left(S_k^2 \rightarrow S_{k+1}^1\right)+\bar{\beta}_{k+1}\left(S_{k+1}^1\right)$$
$$Z_k^d\left(S_k^3 \rightarrow S_{k+1}^5\right)=\bar{\alpha}_k\left(S_k^3\right)+\bar{\gamma}_{k+1}\left(S_k^3 \rightarrow S_{k+1}^5\right)+\bar{\beta}_{k+1}\left(S_{k+1}^5\right)$$
$$Z_k^e\left(S_k^4 \rightarrow S_{k+1}^6\right)=\bar{\alpha}_k\left(S_k^4\right)+\bar{\gamma}_{k+1}\left(S_k^4 \rightarrow S_{k+1}^6\right)+\bar{\beta}_{k+1}\left(S_{k+1}^6\right)$$
$$Z_k^f\left(S_k^5 \rightarrow S_{k+1}^2\right)=\bar{\alpha}_k\left(S_k^5\right)+\bar{\gamma}_{k+1}\left(S_k^5 \rightarrow S_{k+1}^2\right)+\bar{\beta}_{k+1}\left(S_{k+1}^2\right)$$
$$Z_k^g\left(S_k^6 \rightarrow S_{k+1}^7\right)=\bar{\alpha}_k\left(S_k^6\right)+\bar{\gamma}_{k+1}\left(S_k^6 \rightarrow S_{k+1}^7\right)+\bar{\beta}_{k+1}\left(S_{k+1}^7\right)$$
$$Z_k^h\left(S_k^7 \rightarrow S_{k+1}^3\right)=\bar{\alpha}_k\left(S_k^7\right)+\bar{\gamma}_{k+1}\left(S_k^7 \rightarrow S_{k+1}^3\right)+\bar{\beta}_{k+1}\left(S_{k+1}^3\right)$$

$$t_k(0,0)=\max\left\{\begin{array}{l} Z_k^a\left(S_k^0 \rightarrow S_{k+1}^0\right), Z_k^b\left(S_k^1 \rightarrow S_{k+1}^4\right), Z_k^c\left(S_k^2 \rightarrow S_{k+1}^1\right), Z_k^d\left(S_k^3 \rightarrow S_{k+1}^5\right), \\ Z_k^e\left(S_k^4 \rightarrow S_{k+1}^6\right), Z_k^f\left(S_k^5 \rightarrow S_{k+1}^2\right), Z_k^g\left(S_k^6 \rightarrow S_{k+1}^7\right), Z_k^h\left(S_k^7 \rightarrow S_{k+1}^3\right) \end{array}\right\}$$

Turbo Decoder

- Max-Log MAP – LTE

$$\gamma_{ij} = \mathrm{V}(X_k)X(i,j) + \Lambda^i(Z_k)Z(i,j),$$

$$\mathrm{V}(X_k) = \begin{cases} \mathrm{V}_1(X_k) = \Lambda^i(X_k) + \mathrm{W}(X_k), \text{for SISO 1} \\ \mathrm{V}_2(X_k^{'}) = \mathrm{IL}\{\Lambda_1^o(X_k) - \mathrm{W}(X_k)\}, \text{for SISO 2} \end{cases}$$

$$\Lambda^i(Z_k) = \begin{cases} \Lambda^i(Z_k), \text{for SISO 1} \\ \Lambda^i(Z_k^{'}), \text{for SISO 2} \end{cases}$$

$$\gamma_0 = 0$$
$$\gamma_1 = \mathrm{V}(X_k)$$
$$\gamma_2 = \Lambda^i(Z_k)$$
$$\gamma_3 = \mathrm{V}(X_k) + \Lambda^i(Z_k)$$

$$\gamma_{00} = \gamma_0;\ \gamma_{04} = \gamma_3$$
$$\gamma_{10} = \gamma_3;\ \gamma_{14} = \gamma_0$$
$$\gamma_{21} = \gamma_1;\ \gamma_{25} = \gamma_2$$
$$\gamma_{31} = \gamma_2;\ \gamma_{35} = \gamma_1$$
$$\gamma_{42} = \gamma_2;\ \gamma_{46} = \gamma_1$$
$$\gamma_{52} = \gamma_1;\ \gamma_{56} = \gamma_2$$
$$\gamma_{63} = \gamma_3;\ \gamma_{67} = \gamma_0$$
$$\gamma_{73} = \gamma_0;\ \gamma_{77} = \gamma_3$$

$$\hat{\alpha}_k(S_0) = \max\{(\alpha_{k-1}(S_0) + \gamma_{00}), (\alpha_{k-1}(S_1) + \gamma_{10})\}$$
$$\hat{\alpha}_k(S_1) = \max\{(\alpha_{k-1}(S_2) + \gamma_{21}), (\alpha_{k-1}(S_3) + \gamma_{31})\}$$
$$\hat{\alpha}_k(S_2) = \max\{(\alpha_{k-1}(S_4) + \gamma_{42}), (\alpha_{k-1}(S_5) + \gamma_{52})\}$$
$$\hat{\alpha}_k(S_3) = \max\{(\alpha_{k-1}(S_6) + \gamma_{63}), (\alpha_{k-1}(S_7) + \gamma_{73})\}$$
$$\hat{\alpha}_k(S_4) = \max\{(\alpha_{k-1}(S_0) + \gamma_{04}), (\alpha_{k-1}(S_1) + \gamma_{14})\}$$
$$\hat{\alpha}_k(S_5) = \max\{(\alpha_{k-1}(S_2) + \gamma_{25}), (\alpha_{k-1}(S_3) + \gamma_{35})\}$$
$$\hat{\alpha}_k(S_6) = \max\{(\alpha_{k-1}(S_4) + \gamma_{46}), (\alpha_{k-1}(S_5) + \gamma_{56})\}$$
$$\hat{\alpha}_k(S_7) = \max\{(\alpha_{k-1}(S_6) + \gamma_{67}), (\alpha_{k-1}(S_7) + \gamma_{77})\}$$

$$\hat{\beta}_k(S_0) = \max\{(\beta_{k+1}(S_0) + \gamma_{00}), (\beta_{k+1}(S_4) + \gamma_{04})\}$$
$$\hat{\beta}_k(S_1) = \max\{(\beta_{k+1}(S_0) + \gamma_{10}), (\beta_{k+1}(S_4) + \gamma_{14})\}$$
$$\hat{\beta}_k(S_2) = \max\{(\beta_{k+1}(S_1) + \gamma_{21}), (\beta_{k+1}(S_5) + \gamma_{25})\}$$
$$\hat{\beta}_k(S_3) = \max\{(\beta_{k+1}(S_1) + \gamma_{31}), (\beta_{k+1}(S_5) + \gamma_{35})\}$$
$$\hat{\beta}_k(S_4) = \max\{(\beta_{k+1}(S_2) + \gamma_{42}), (\beta_{k+1}(S_6) + \gamma_{46})\}$$
$$\hat{\beta}_k(S_5) = \max\{(\beta_{k+1}(S_2) + \gamma_{52}), (\beta_{k+1}(S_6) + \gamma_{56})\}$$
$$\hat{\beta}_k(S_6) = \max\{(\beta_{k+1}(S_3) + \gamma_{63}), (\beta_{k+1}(S_7) + \gamma_{67})\}$$
$$\hat{\beta}_k(S_7) = \max\{(\beta_{k+1}(S_3) + \gamma_{73}), (\beta_{k+1}(S_7) + \gamma_{77})\}$$

TURBO DECODER

- Max-Log MAP – LTE

$$\lambda_k^0(0,0)=\alpha_{k-1}(S_0)+\gamma_{00}+\beta_k(S_0)$$
$$\lambda_k^0(1,4)=\alpha_{k-1}(S_1)+\gamma_{14}+\beta_k(S_4)$$
$$\lambda_k^0(2,5)=\alpha_{k-1}(S_2)+\gamma_{25}+\beta_k(S_5)$$
$$\lambda_k^0(3,1)=\alpha_{k-1}(S_3)+\gamma_{31}+\beta_k(S_1)$$
$$\lambda_k^0(4,2)=\alpha_{k-1}(S_4)+\gamma_{42}+\beta_k(S_2)$$
$$\lambda_k^0(5,6)=\alpha_{k-1}(S_5)+\gamma_{56}+\beta_k(S_6)$$
$$\lambda_k^0(6,7)=\alpha_{k-1}(S_6)+\gamma_{67}+\beta_k(S_7)$$
$$\lambda_k^0(7,3)=\alpha_{k-1}(S_7)+\gamma_{73}+\beta_k(S_3)$$

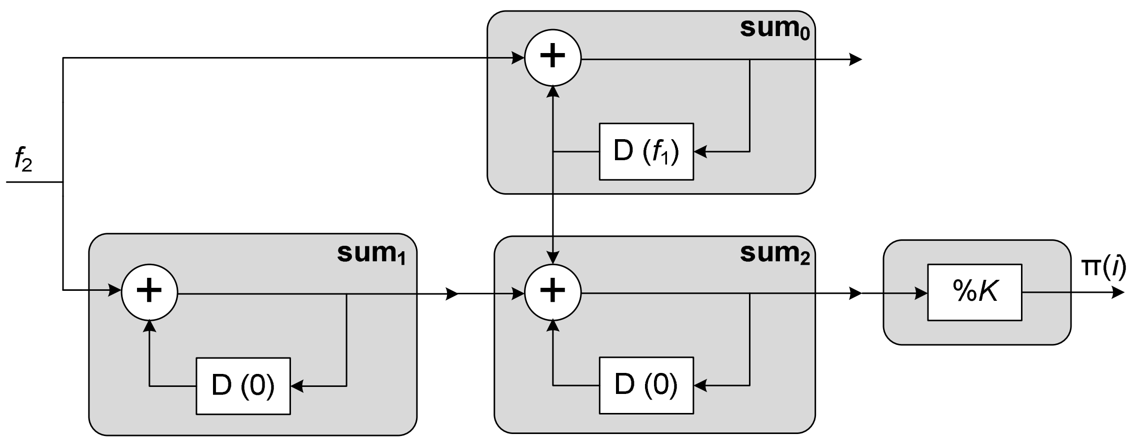
$$\lambda_k^1(0,4)=\alpha_{k-1}(S_0)+\gamma_{04}+\beta_k(S_4)$$
$$\lambda_k^1(1,0)=\alpha_{k-1}(S_1)+\gamma_{10}+\beta_k(S_0)$$
$$\lambda_k^1(2,1)=\alpha_{k-1}(S_2)+\gamma_{21}+\beta_k(S_1)$$
$$\lambda_k^1(3,5)=\alpha_{k-1}(S_3)+\gamma_{35}+\beta_k(S_5)$$
$$\lambda_k^1(4,6)=\alpha_{k-1}(S_4)+\gamma_{46}+\beta_k(S_6)$$
$$\lambda_k^1(5,2)=\alpha_{k-1}(S_5)+\gamma_{52}+\beta_k(S_2)$$
$$\lambda_k^1(6,3)=\alpha_{k-1}(S_6)+\gamma_{63}+\beta_k(S_3)$$
$$\lambda_k^1(7,7)=\alpha_{k-1}(S_7)+\gamma_{77}+\beta_k(S_7)$$

$$\Lambda^o(X_k)=\max_{(S_i\to S_j):X_i=1}\{\lambda_k^1(i,j)\}-\max_{(S_i\to S_j):X_i=0}\{\lambda_k^0(i,j)\},$$

TURBO DECODER

- LTE - Quadratic Permutation Polynomial (QPP) interleaver

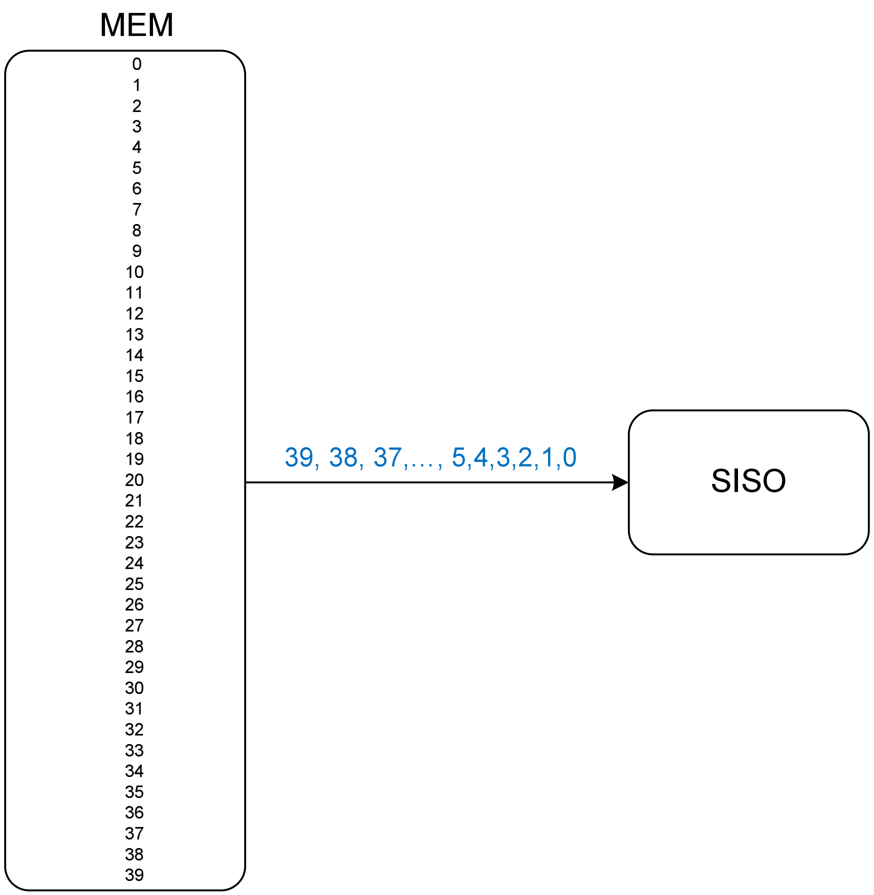
$$\pi(i) = (f_1 \cdot i + f_2 \cdot i^2) \bmod K$$



$$
\begin{aligned}
&\text{sum}_0(0) = f_1, \text{sum}_1(0) = 0, \text{sum}_2(0) = 0 \\
&\pi(0) = 0 \\
&\textit{for } i = 1 : K-1 \\
&\quad \text{sum}_0(i) = \text{sum}_0(i-1) + f_2 \\
&\quad \text{sum}_1(i) = \text{sum}_1(i-1) + f_2 \\
&\quad \text{sum}_2(i) = \text{sum}_2(i-1) + \text{sum}_1(i-1) + \text{sum}_0(i-1) + f_2 \\
&\quad \pi(i) = \text{sum}_2(i) \bmod K \\
&\textit{end}
\end{aligned}
$$

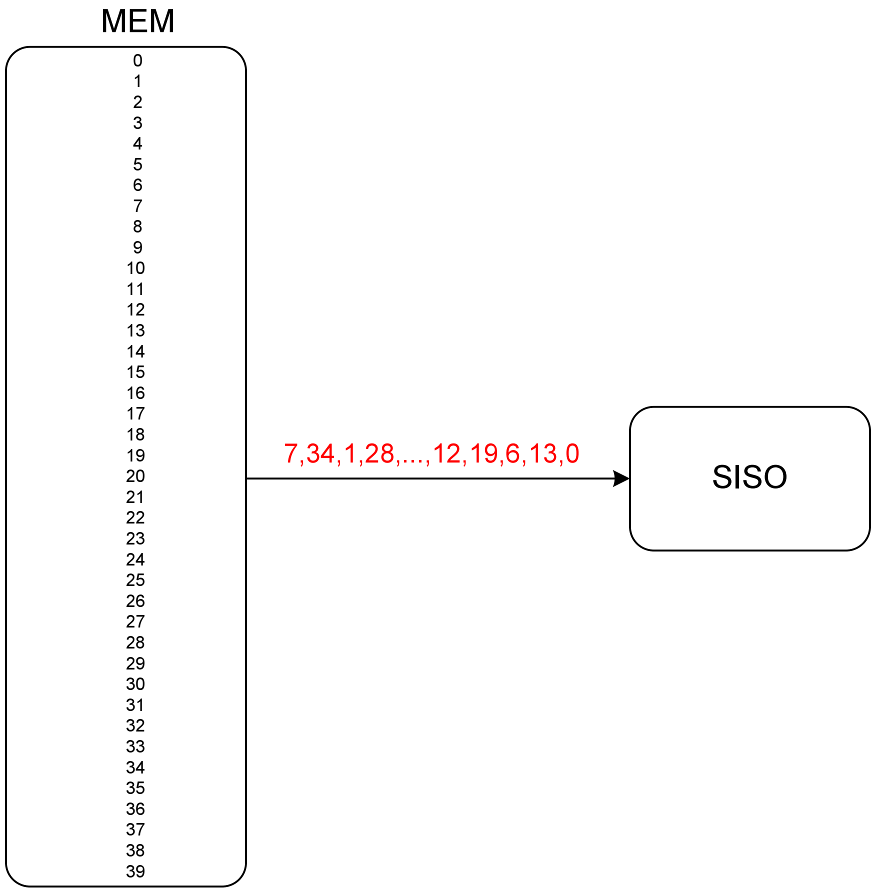
Turbo Decoder

- LTE - Quadratic Permutation Polynomial (QPP) interleaver



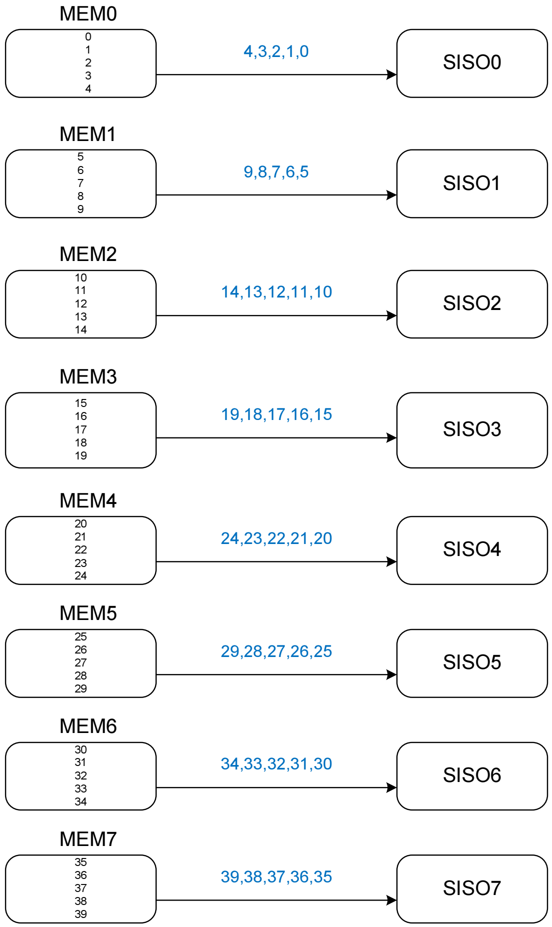
TURBO DECODER

- LTE - Quadratic Permutation Polynomial (QPP) interleaver



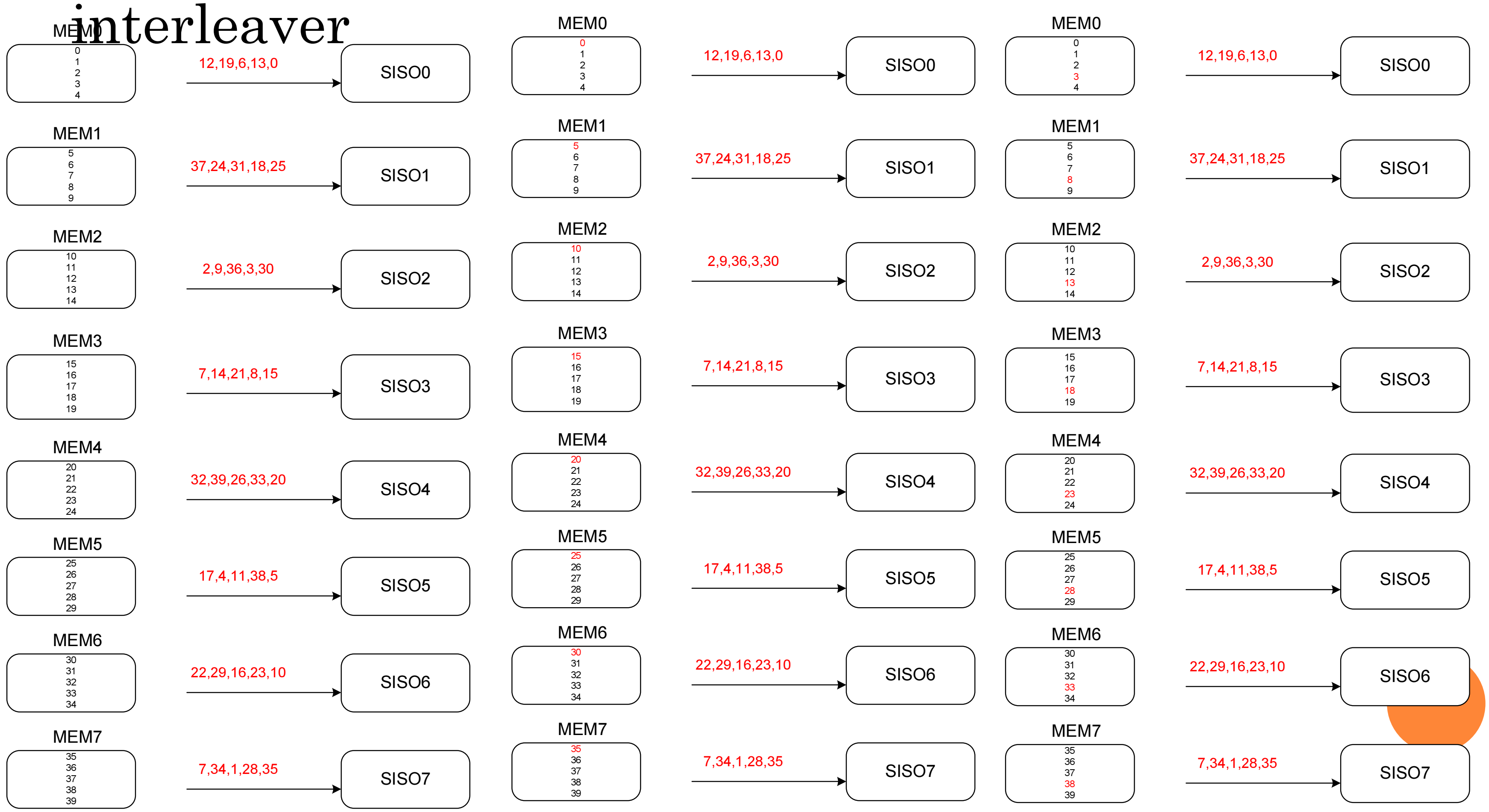
TURBO DECODER

- LTE - Quadratic Permutation Polynomial (QPP) interleaver



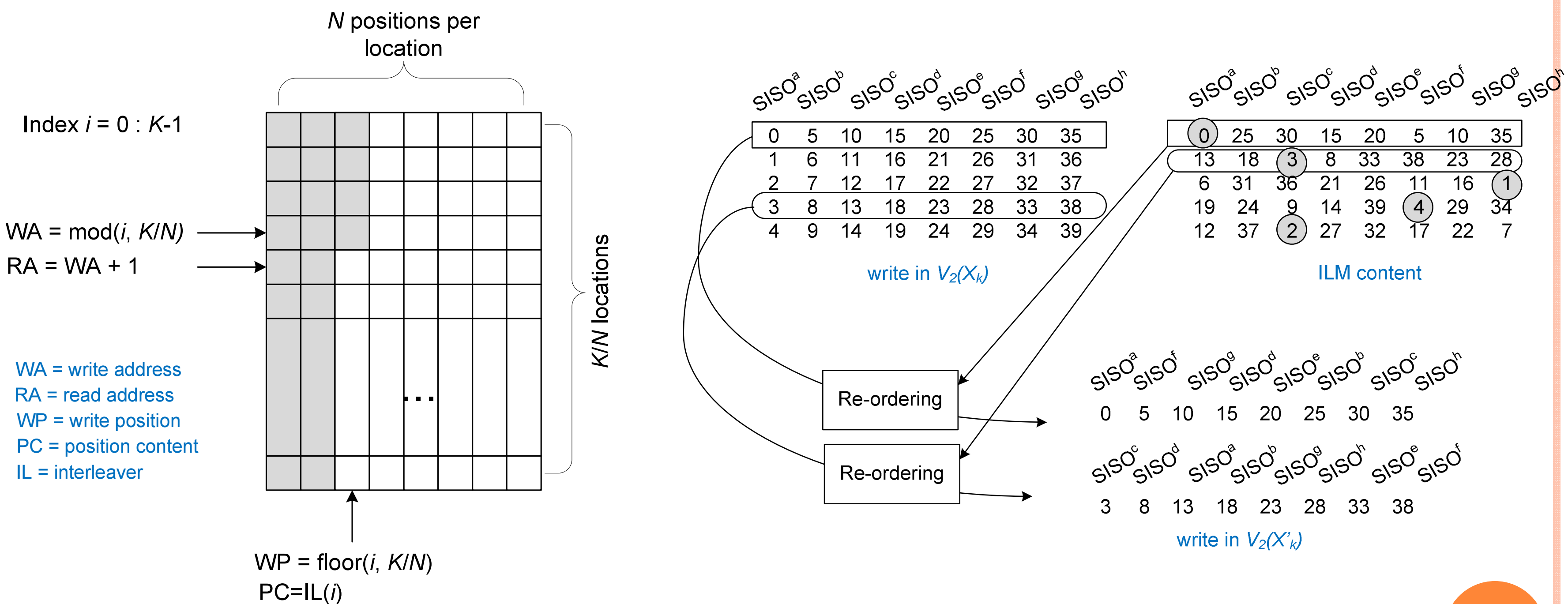
TURBO DECODER

- LTE - Quadratic Permutation Polynomial (QPP) interleaver



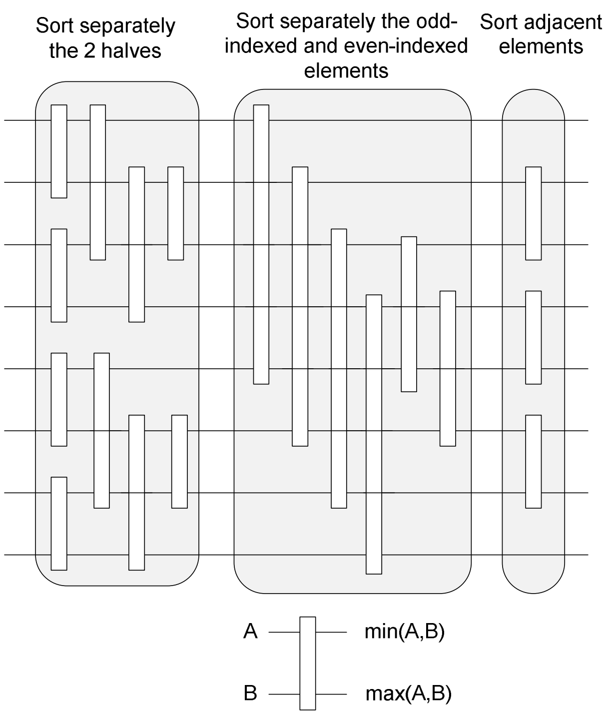
Turbo Decoder

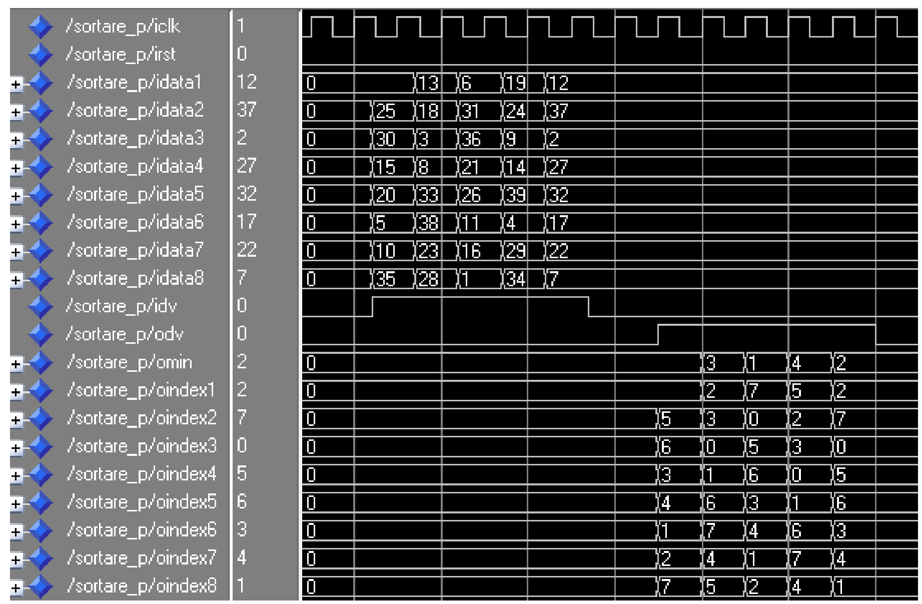
- LTE - Quadratic Permutation Polynomial (QPP) interleaver



TURBO DECODER

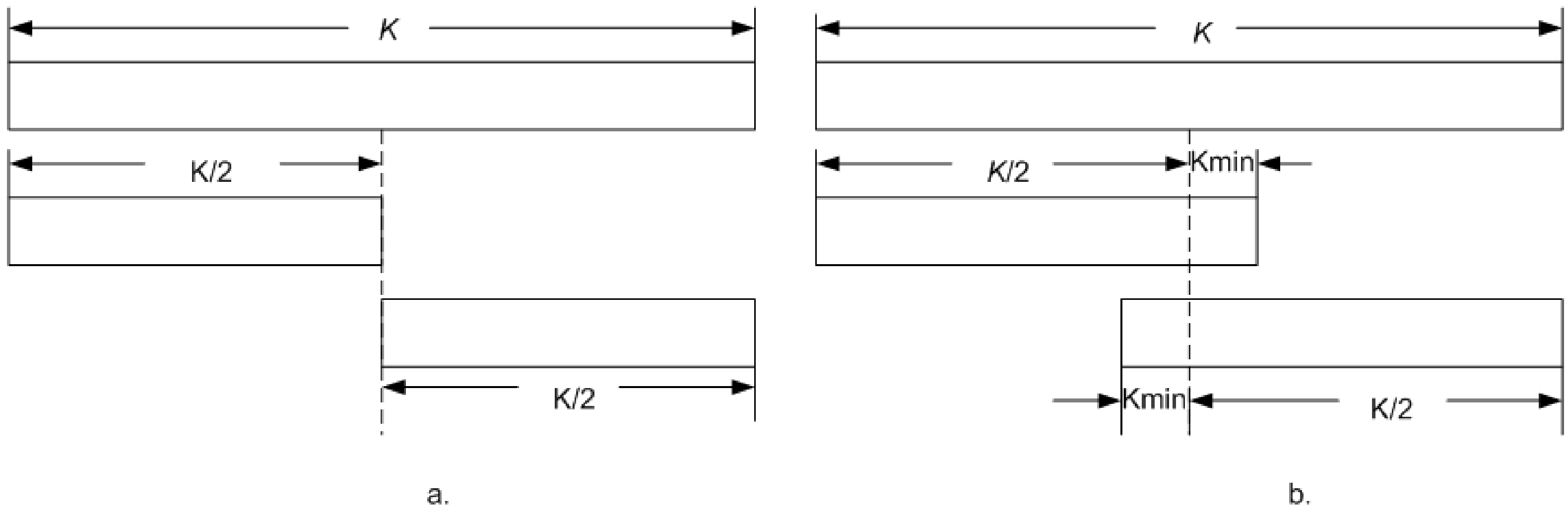
- LTE – reordering unit: even-odd merge sorting





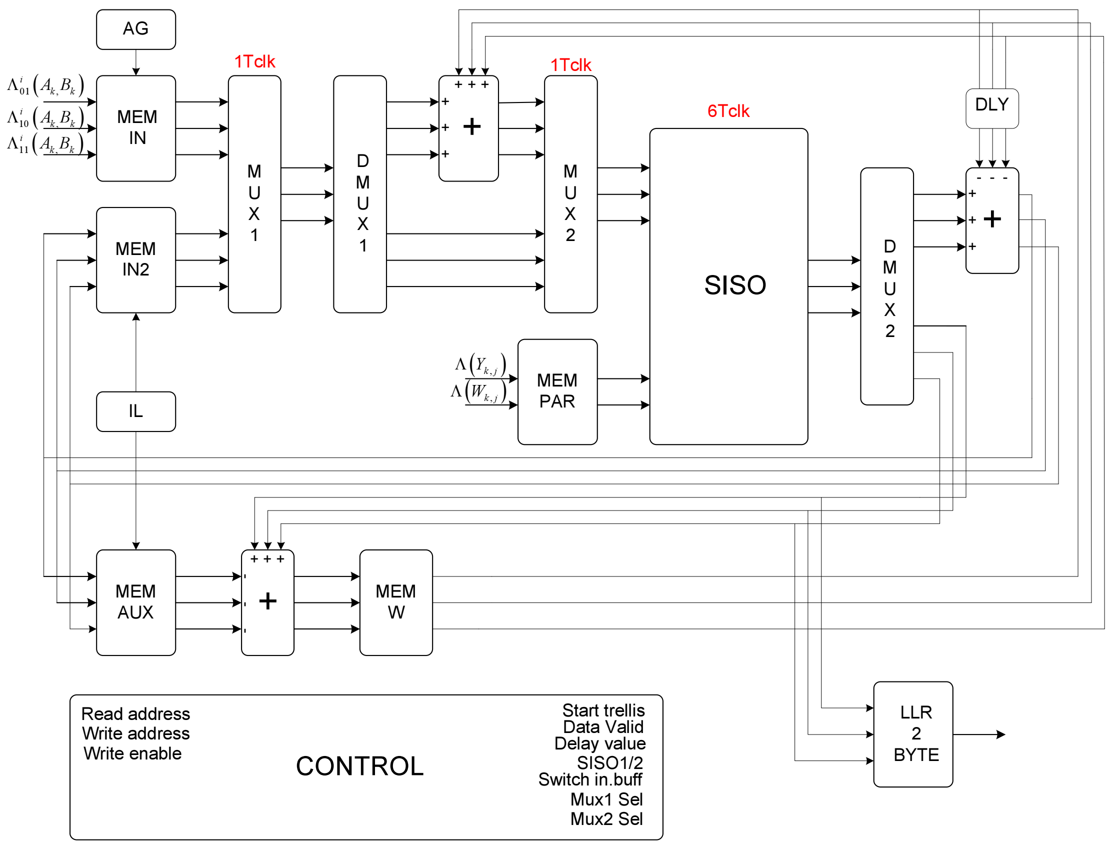
Turbo Decoder

- LTE – split with overlap



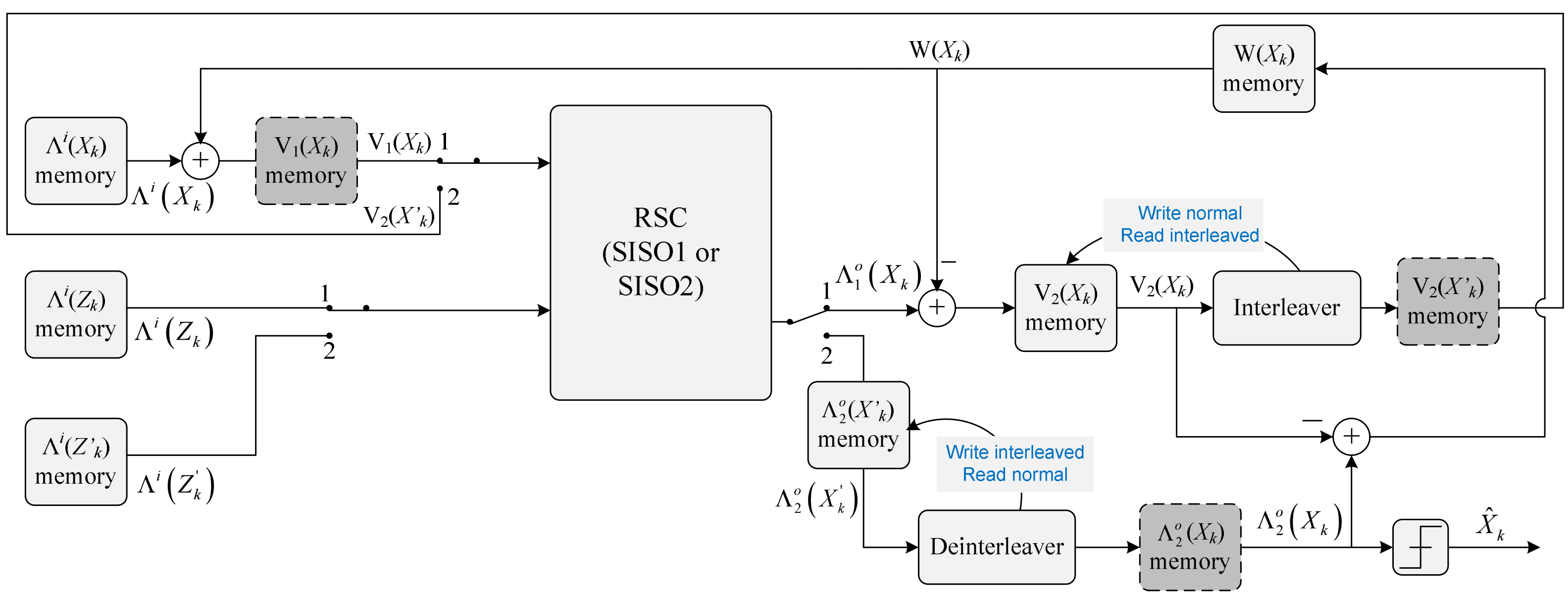
Turbo Decoder

- WiMAX – proposed architecture



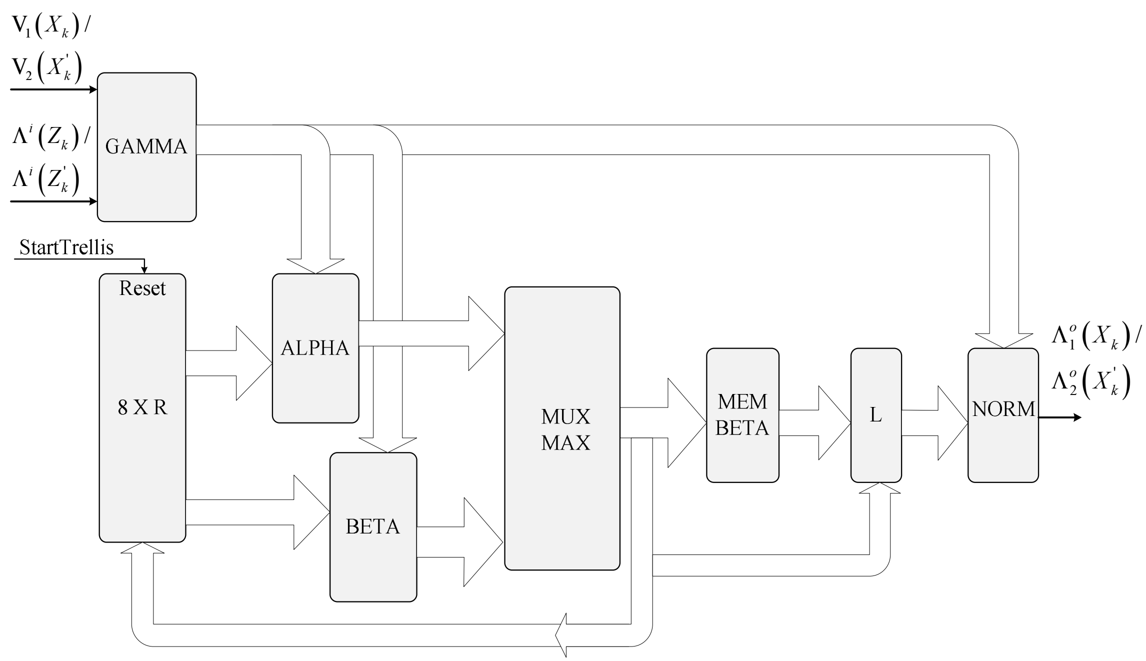
Turbo Decoder

- LTE – proposed architecture



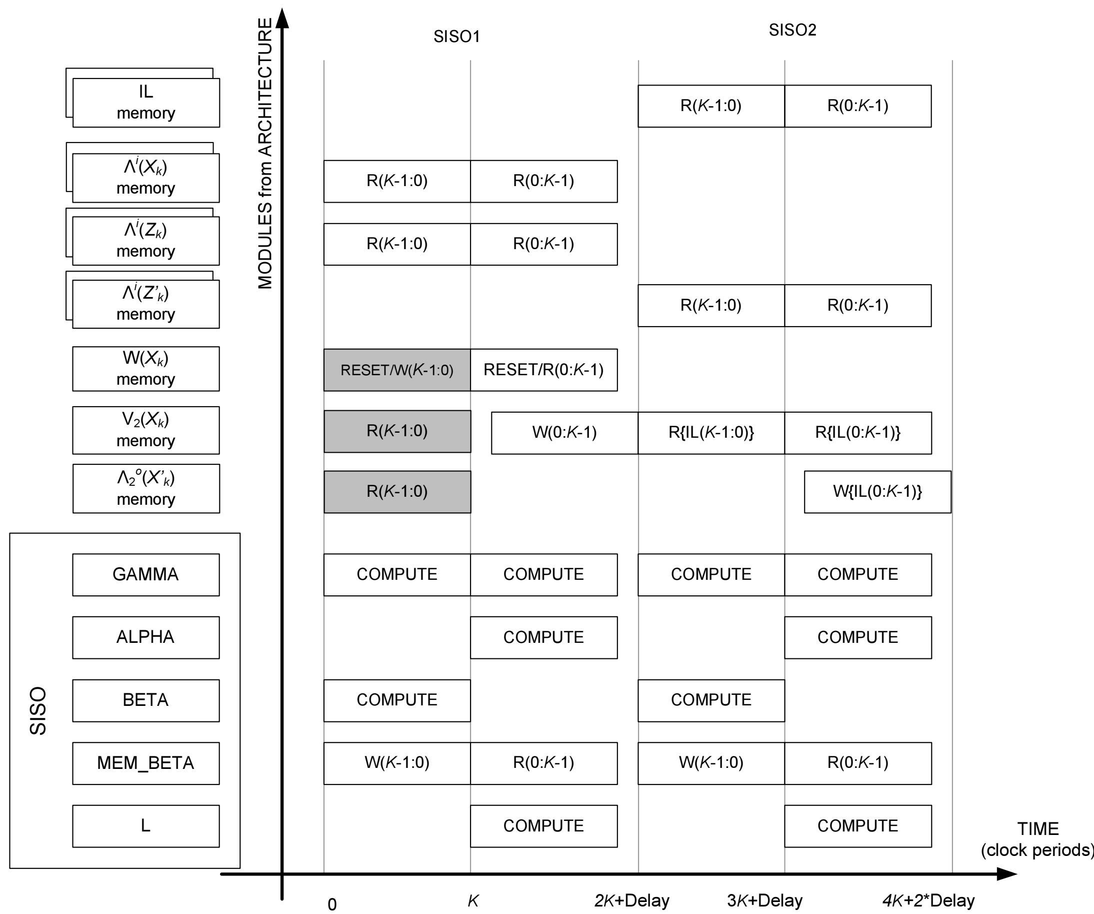
TURBO DECODER

- LTE – SISO proposed architecture

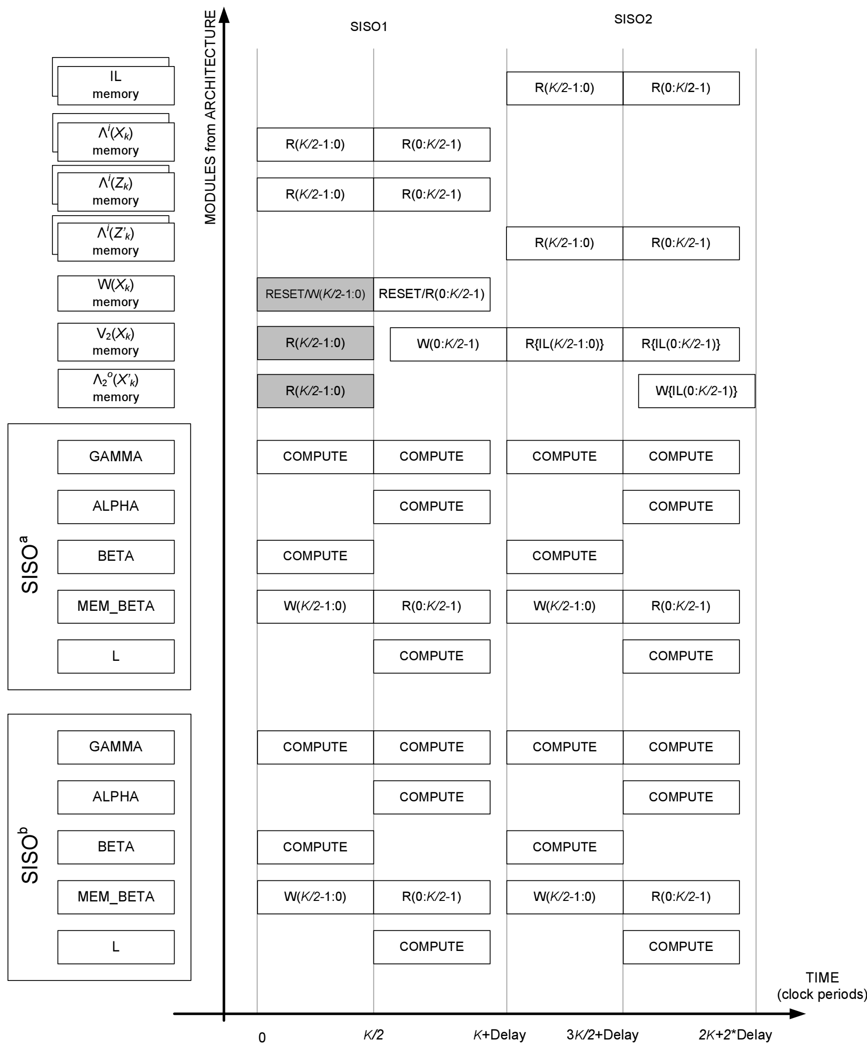


Turbo Decoder

- LTE – timing diagram: serial vs. parallel



Turbo Decoder

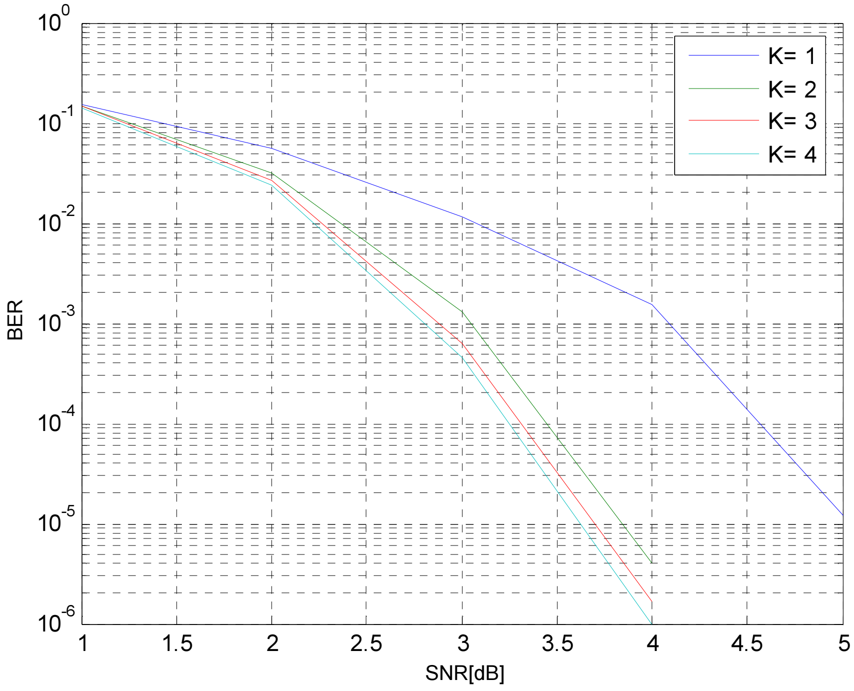


Obtained Results

- WiMAX

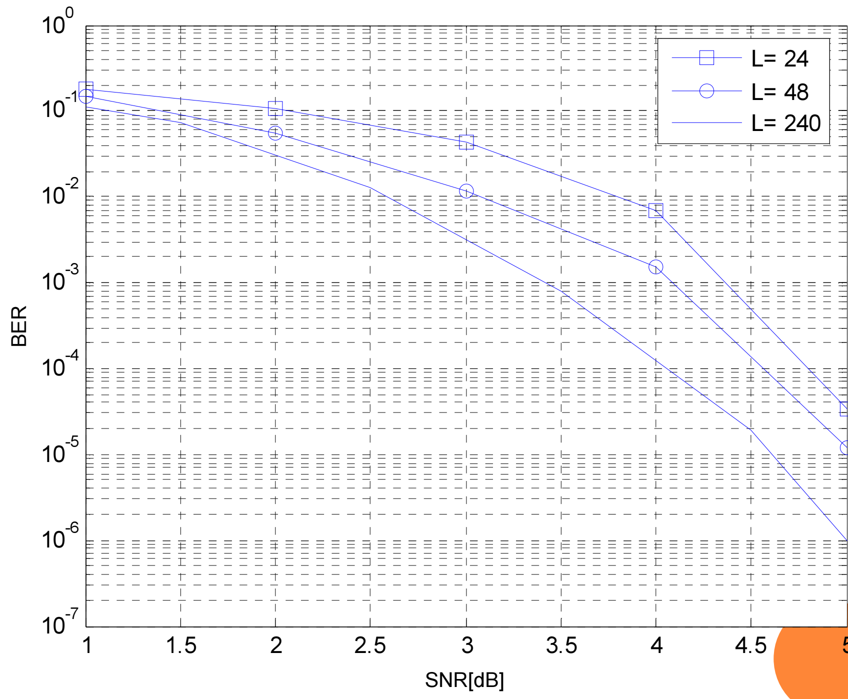
Number of Iterations

L=48, QPSK 1/2



Block Size

K=1, QPSK 1/2

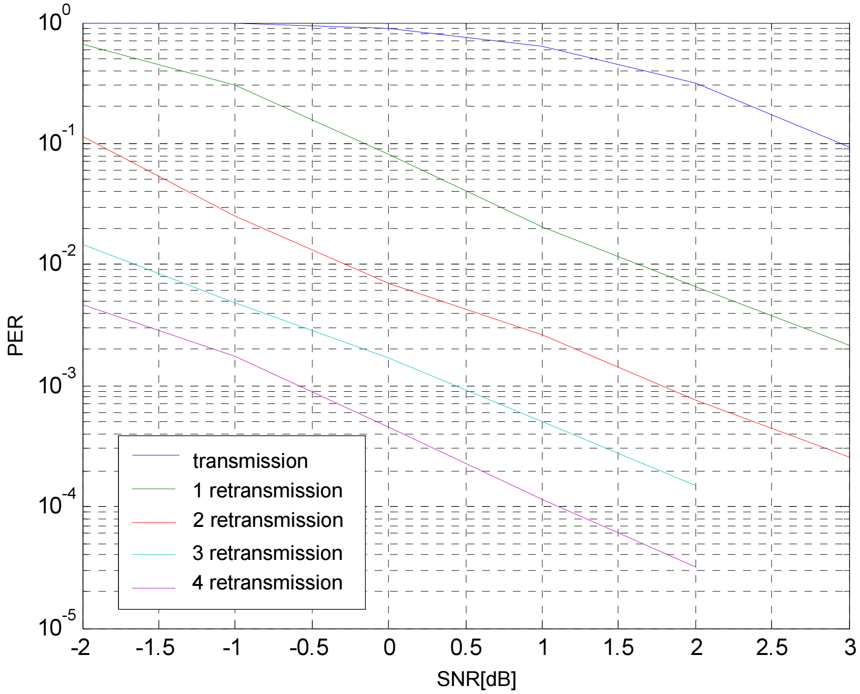


Obtained Results

- WiMAX

Number of Retransmisions

L=24, QPSK ½, K=4

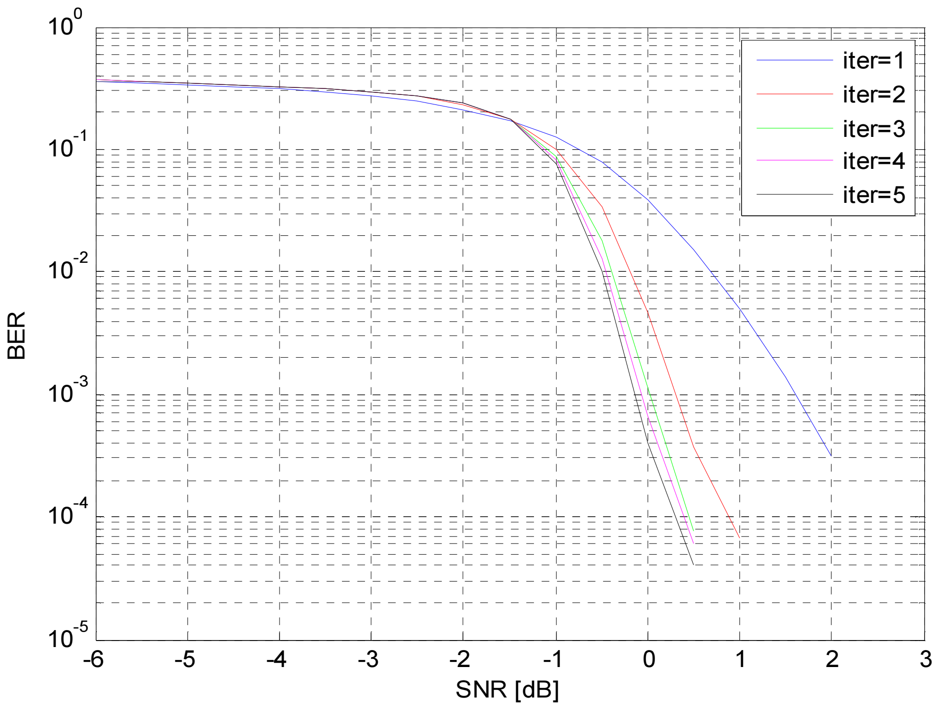


Obtained Results

- LTE

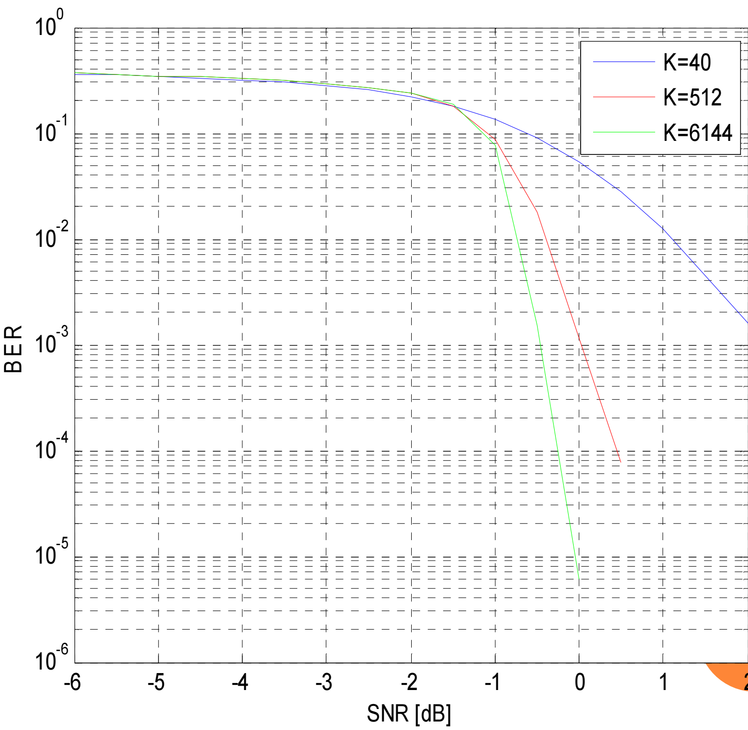
Number of Iterations

L=512, QPSK 1/2



Block Size

K=3, QPSK 1/2

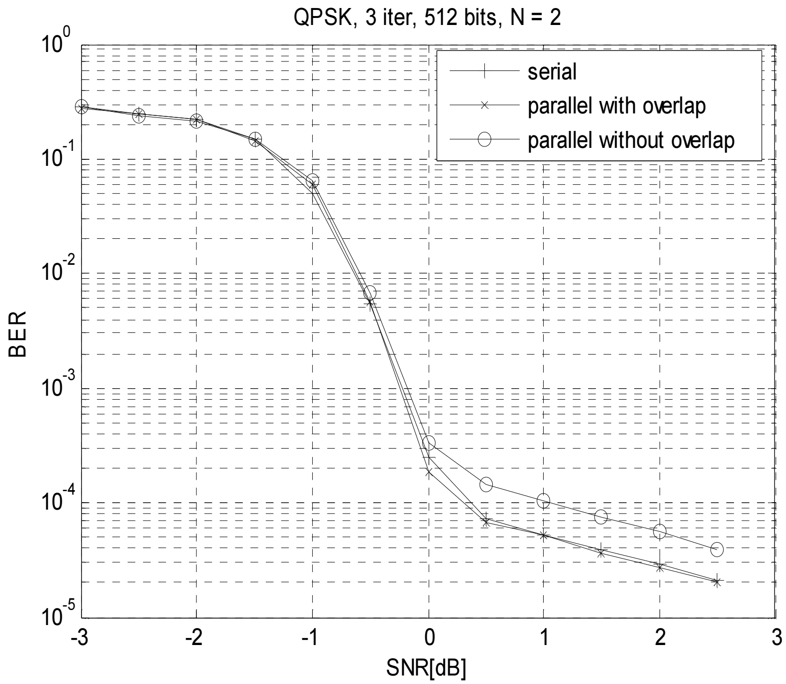


OBTAINED RESULTS

- LTE

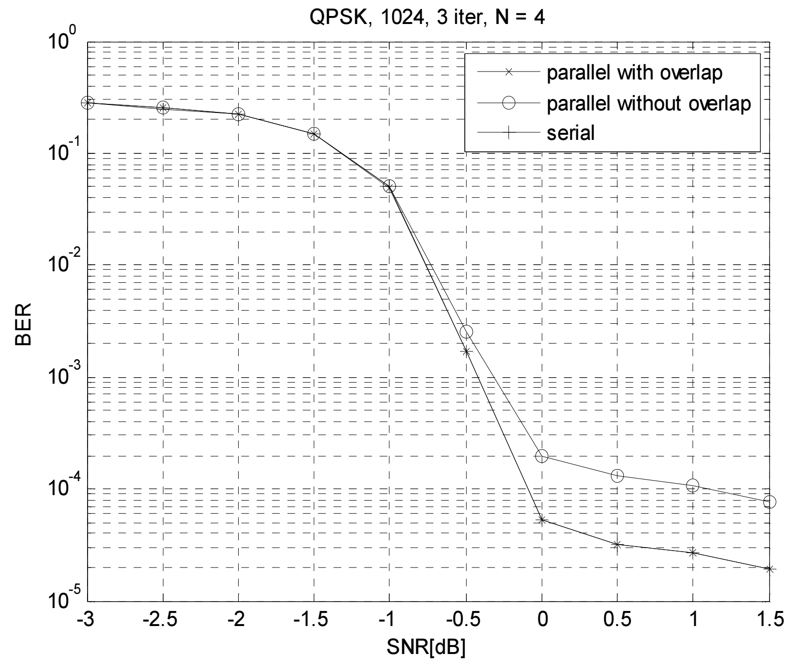
Parallel N=2

L=512, QPSK 1/2, K=3



Parallel N=4

L=1024, QPSK 1/2, K=3



Conclusions

- Efficient Max-Log MAP implementation
- Only one SISO
- All latency-reduction procedures can be applied over the proposed decoding scheme
- Turbo decoding serial architecture adapted for parallel decoding
- Only one interleaver used in the proposed parallel decoding architecture
- Efficient implementation for the interleaver
- Almost identical decoding performances for serial vs. parallel decoding when small overlap accepted

THANK YOU

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