The Role of Infinite Dimensional Adaptive Control Theory in Autonomous Systems

(or When Will SkyNet Take Over the World)

Mark's Autonomous Control Laboratory

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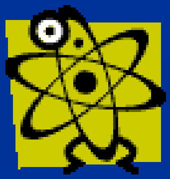
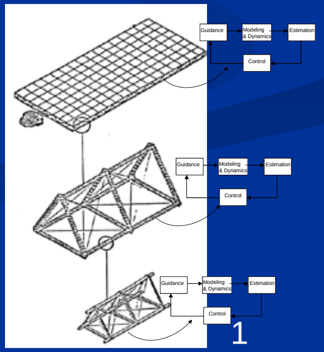
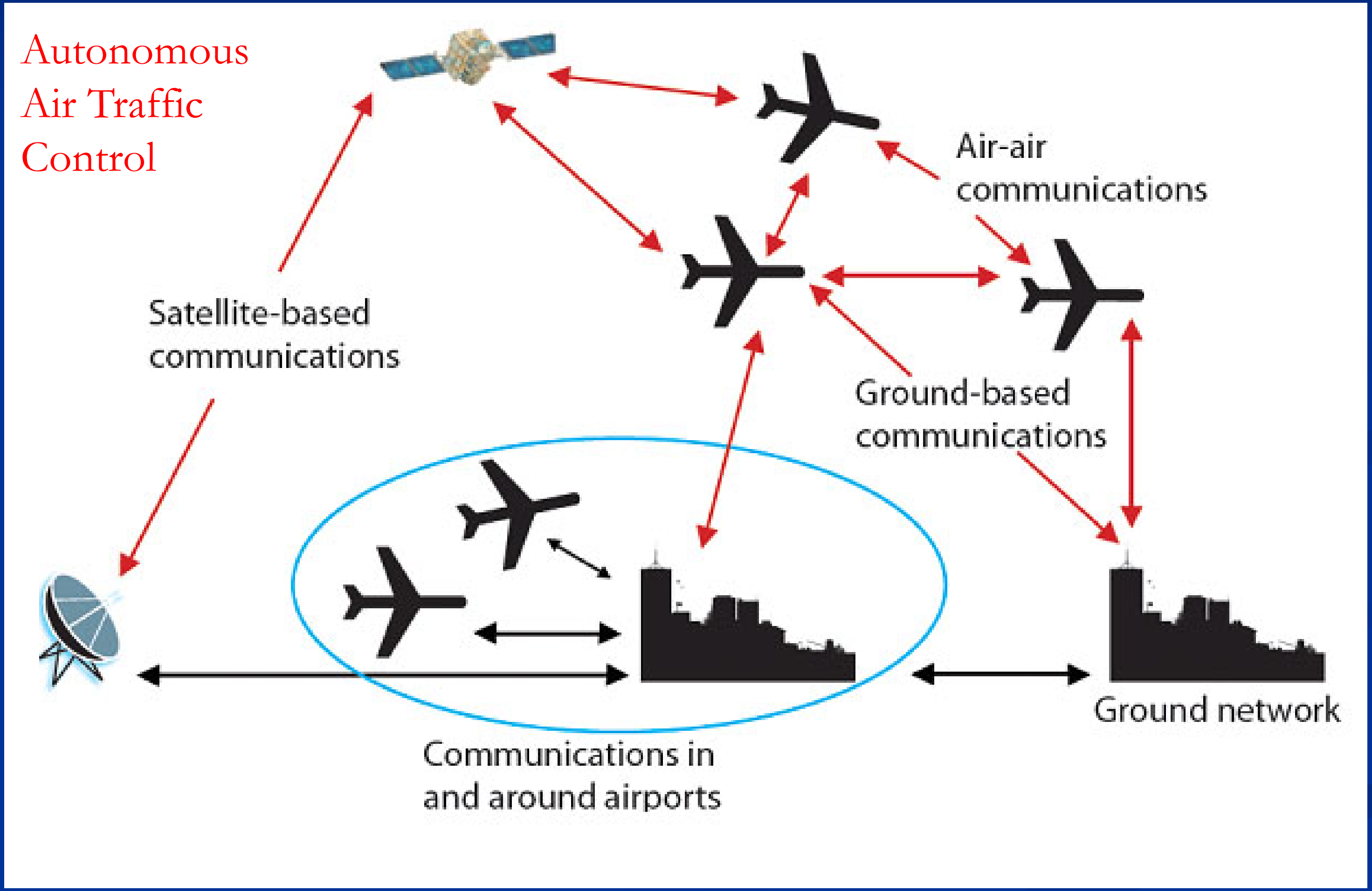




Autonomy:

Greek autonomía =independence

(**philosophy**) the doctrine that the individual human will is or ought to be governed only by its own principles and laws



Will Autonomous Systems Take Over the World?

My brain is a Neural Net Processor

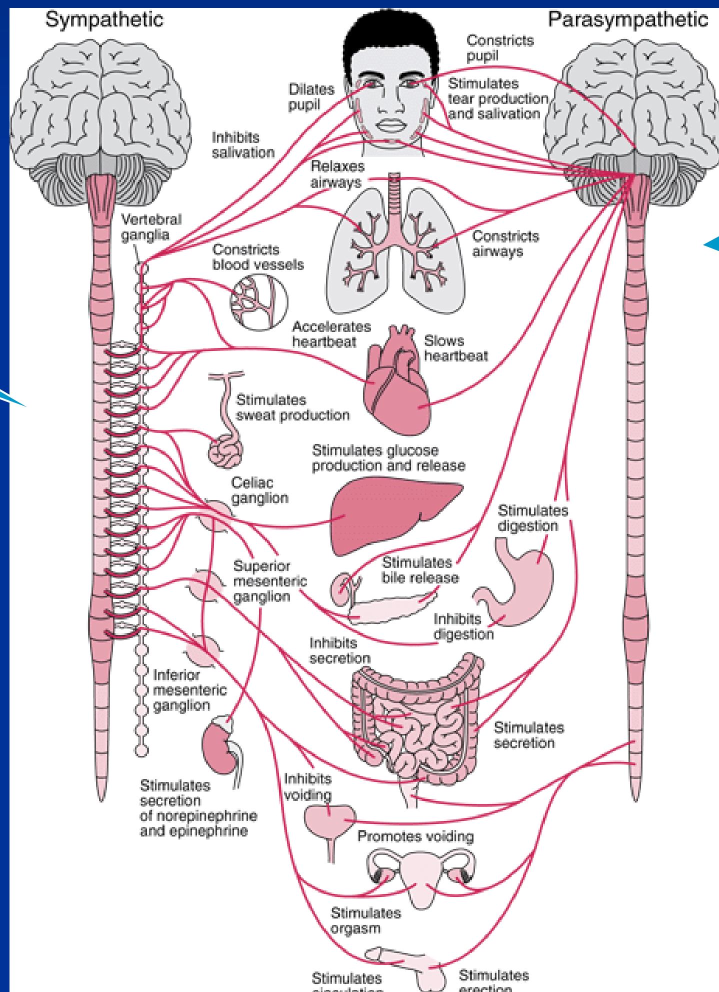
SKYNET
SAC NORAD

Artificial Intelligence Controlled Network Defense Computer System

Adaptive Control Systems

Autonomous Systems vs Autonomic Systems

involuntary or spontaneous



Humans usually do not have to think about the operation of these autonomic systems and so they can do other higher level things, eg sex, murder, presidential elections

F-16 Flexible Structure Model: Fluid-Structure Interaction

Flutter

USAF-Edwards AFB
Flight Test Center

Hypersonic Aircraft X51A Wave Rider

AFRL-
Wright Patterson AFB

6 Minutes at Mach 5.1

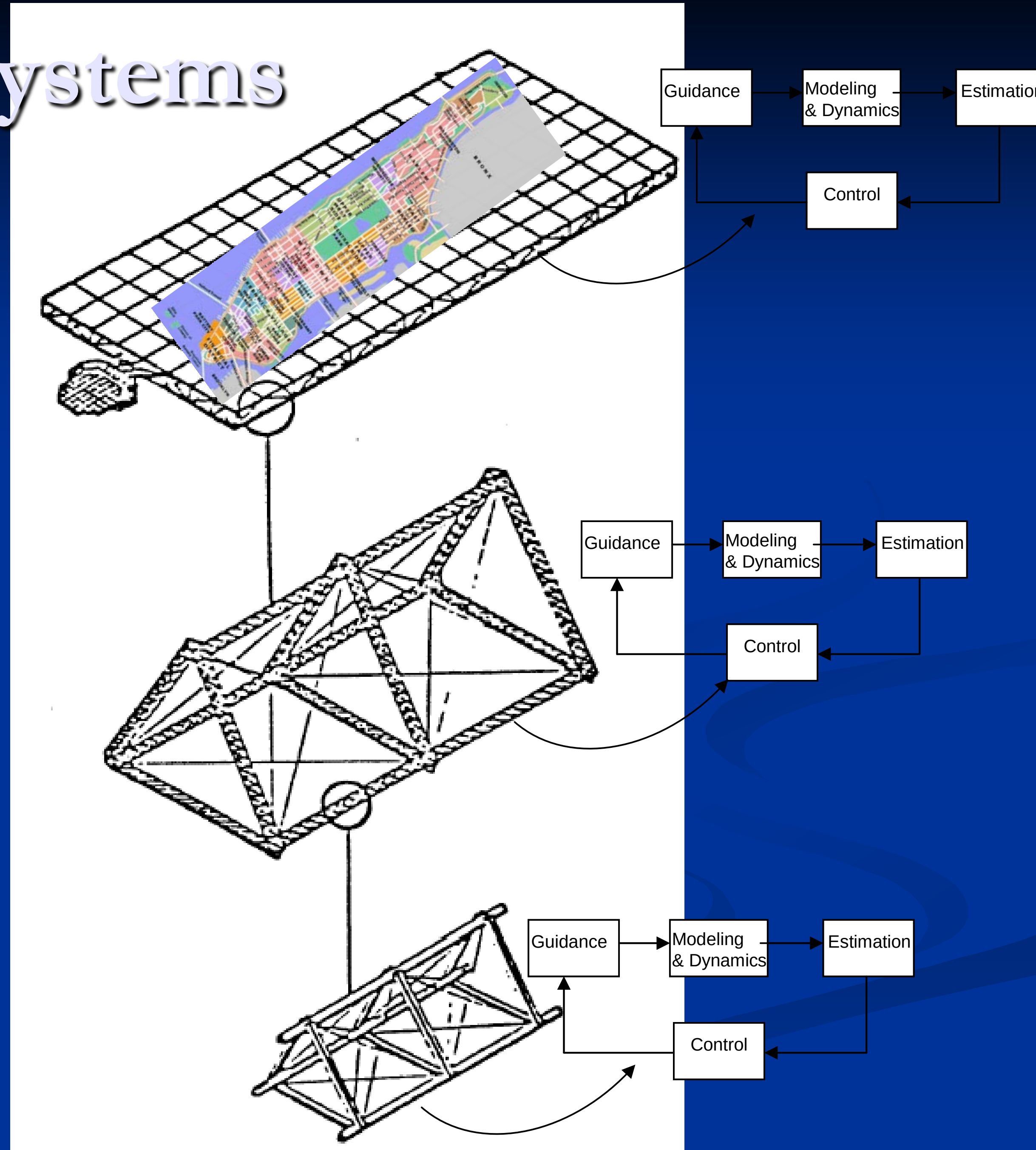
The X-51A WaveRider is an unmanned, autonomous supersonic combustion, ramjet-powered hypersonic flight-test demonstrator for the U.S. Air Force. The X-51A demonstrates a scalable, robust endothermic hydrocarbon-fueled scramjet propulsion system in flight, as well as high temperature materials, airframe/engine integration and other key technologies within the hypersonic range of Mach 4.5 to 6.5.

Evolving Systems

Evolving Systems=
Autonomously
Assembled
Active Structures

Or Self-Assembling
Structures,
which Aspire to a
Higher Purpose;
Cannot be attained
by Components Alone

NASA-JPL



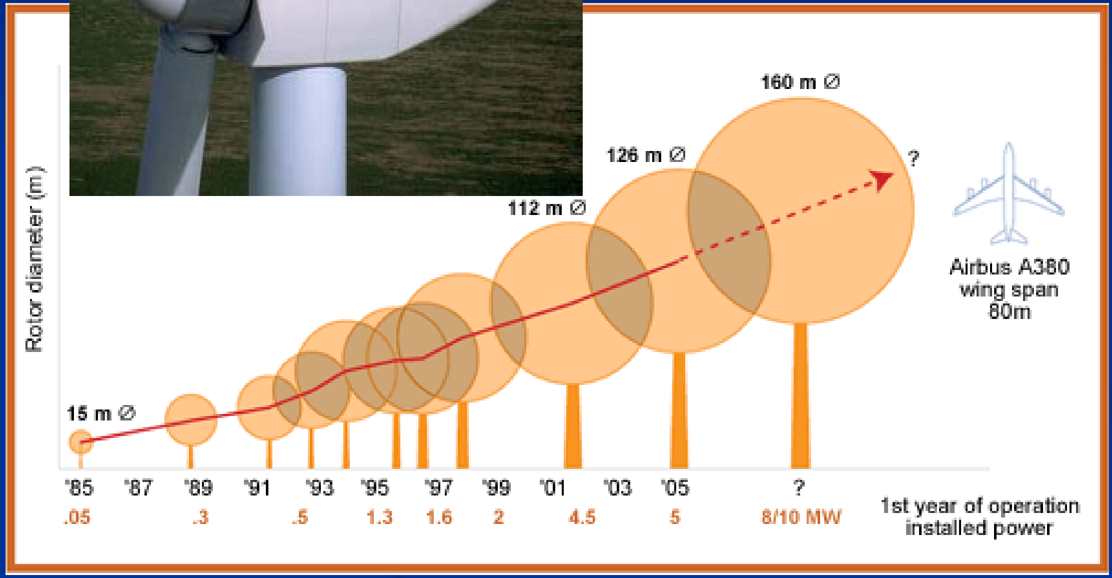
Susan Frost
Intelligent Systems
NASA Ames Research Center

Wind Energy

1979: 40 cents/kWh

2000: 4 - 6 cents/kWh

2006: 3 - 5 cents/kWh



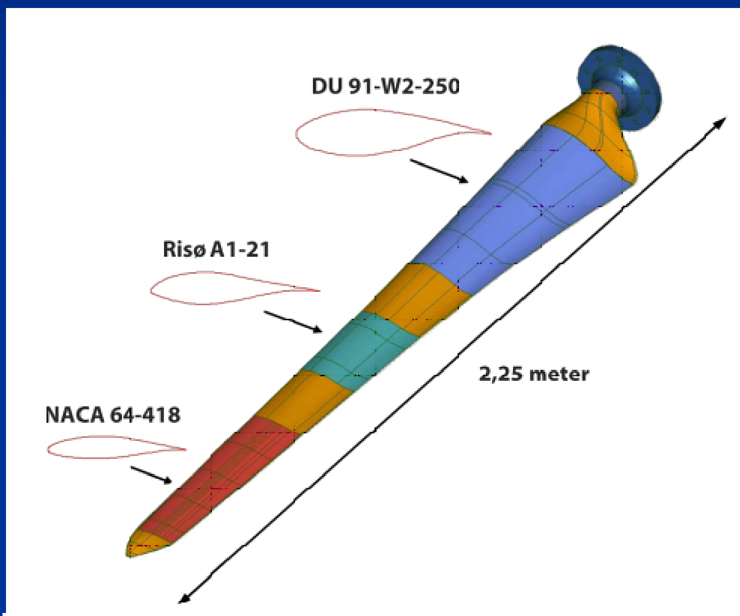
210 MW Lake Benton Wind Farm 4 cents/kWh

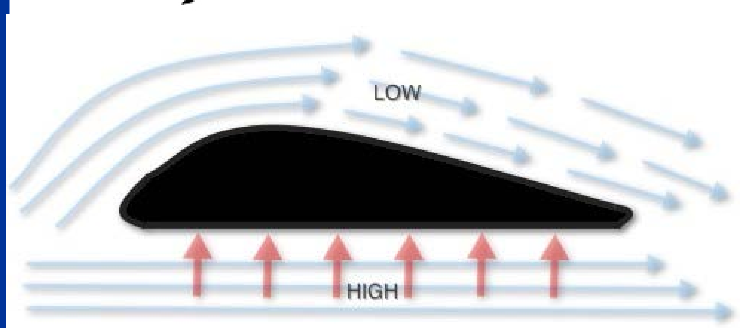
- R & D Advances
- Increased Turbine Size
- Manufacturing Improvements
- Large Wind Farms

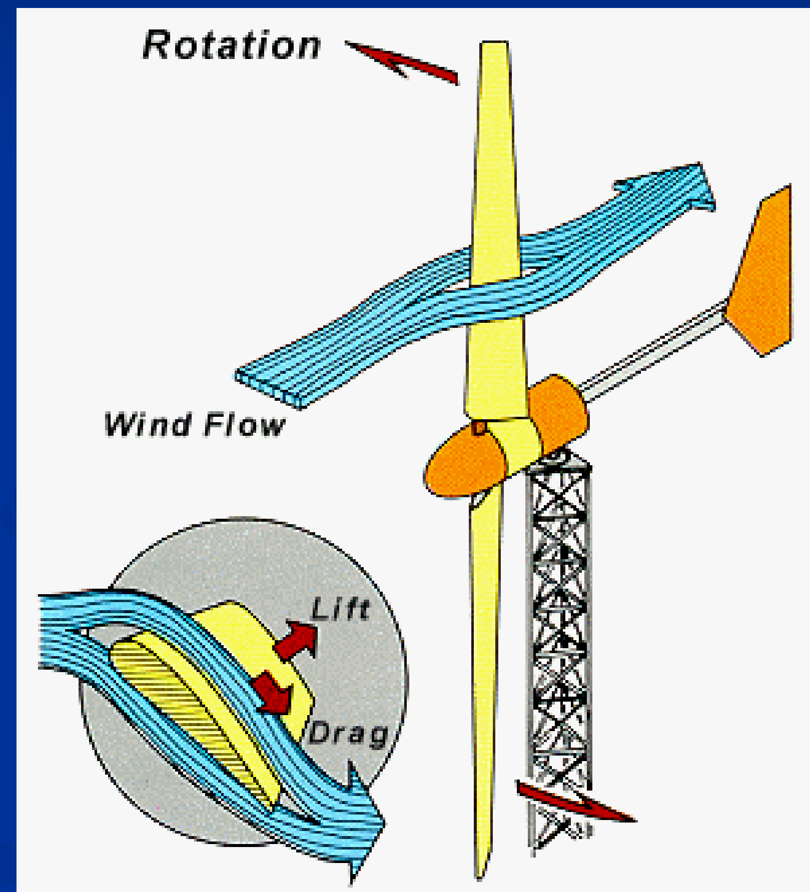
Who Needs Control Anyway?

NREL-NWTC

Flow Control of Wind Turbine Aerodynamics

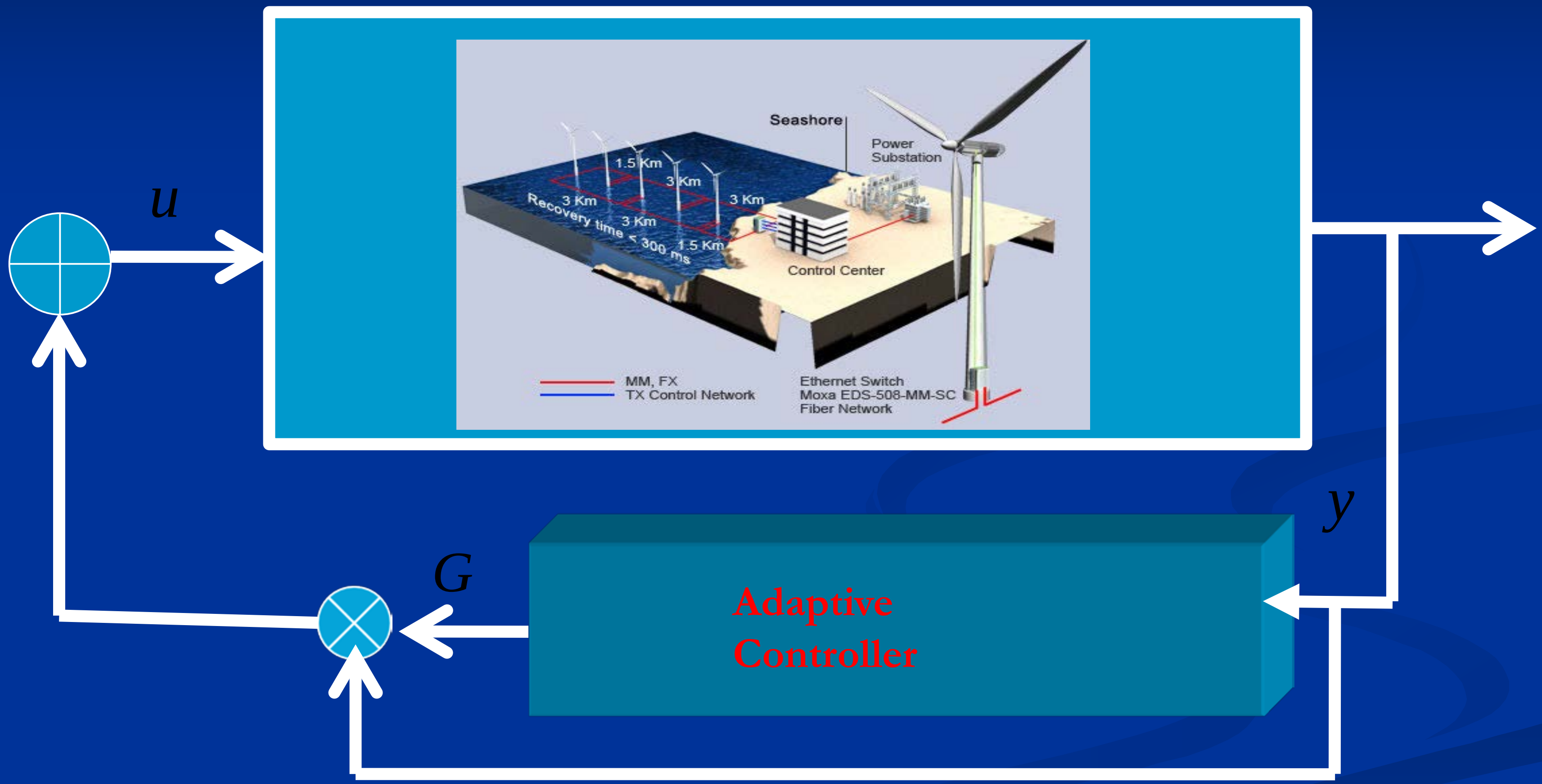




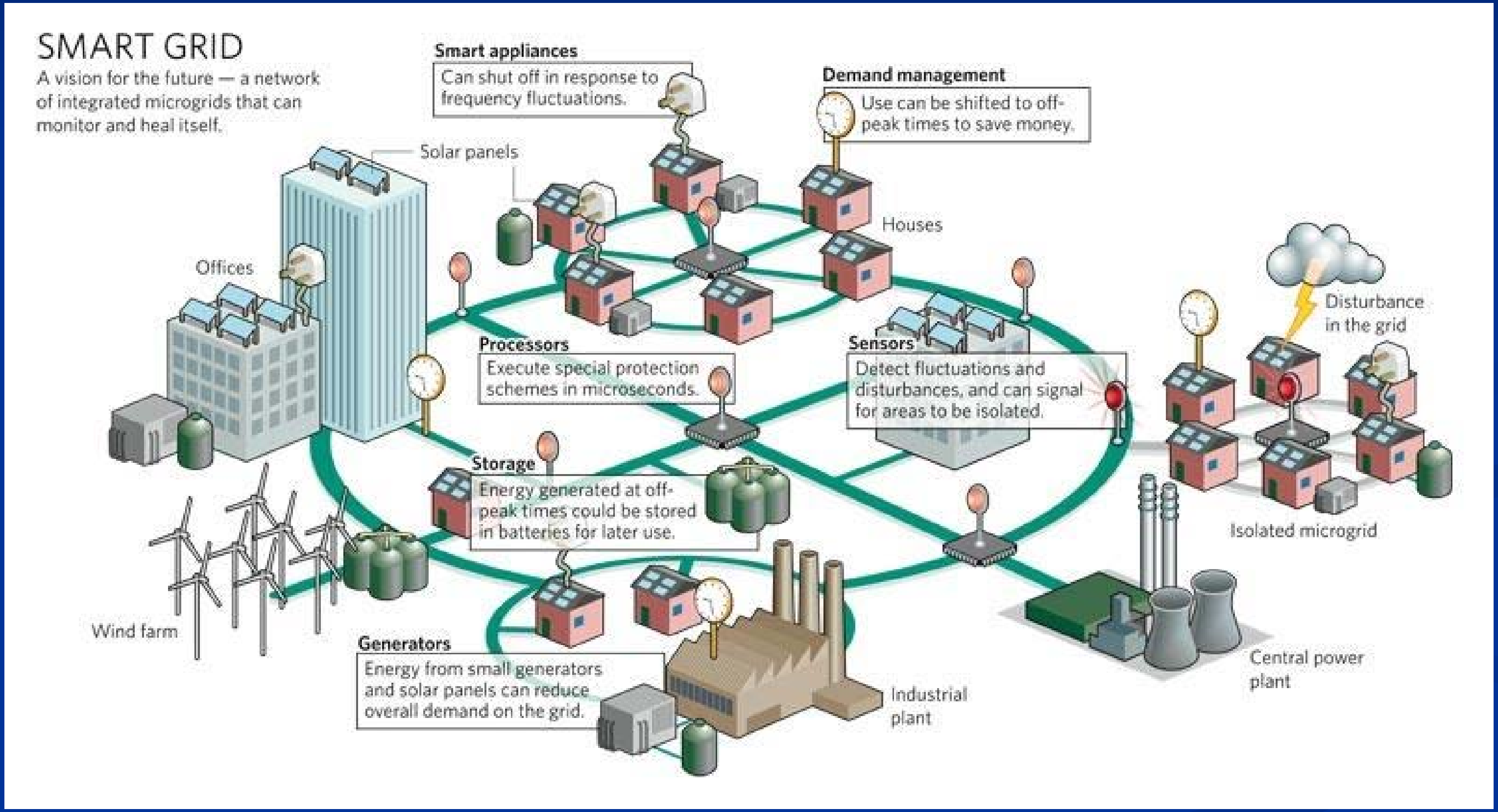


Principles of Wind Turbine Aerodynamic Lift

Fondest Hopes Wildest Dreams: Control the Whole Wind Farm as One Turbine!



Smart Grids: Virtual Interconnecting Forces



"It is surprising how quickly we replace a human operator with an algorithm and call it SMART"

Power System Perturbed with a Wind Farm

- When a wind farm is placed at a distance of α, the perturbed power system becomes :

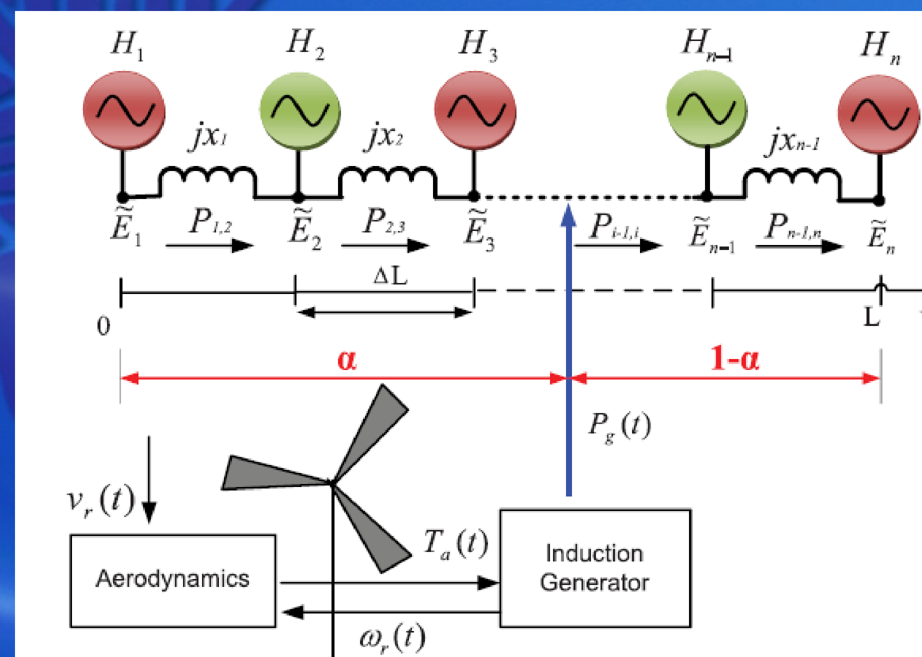
$$\frac{\partial^2 \delta(u,t)}{\partial t^2} + \eta \frac{\partial \delta(u,t)}{\partial t} - \nu^2 \frac{\partial^2 \delta(u,t)}{\partial u^2} = W(u,t)$$

with $W(u,t) = P_g(t)\hat{\delta}(u-\alpha)$

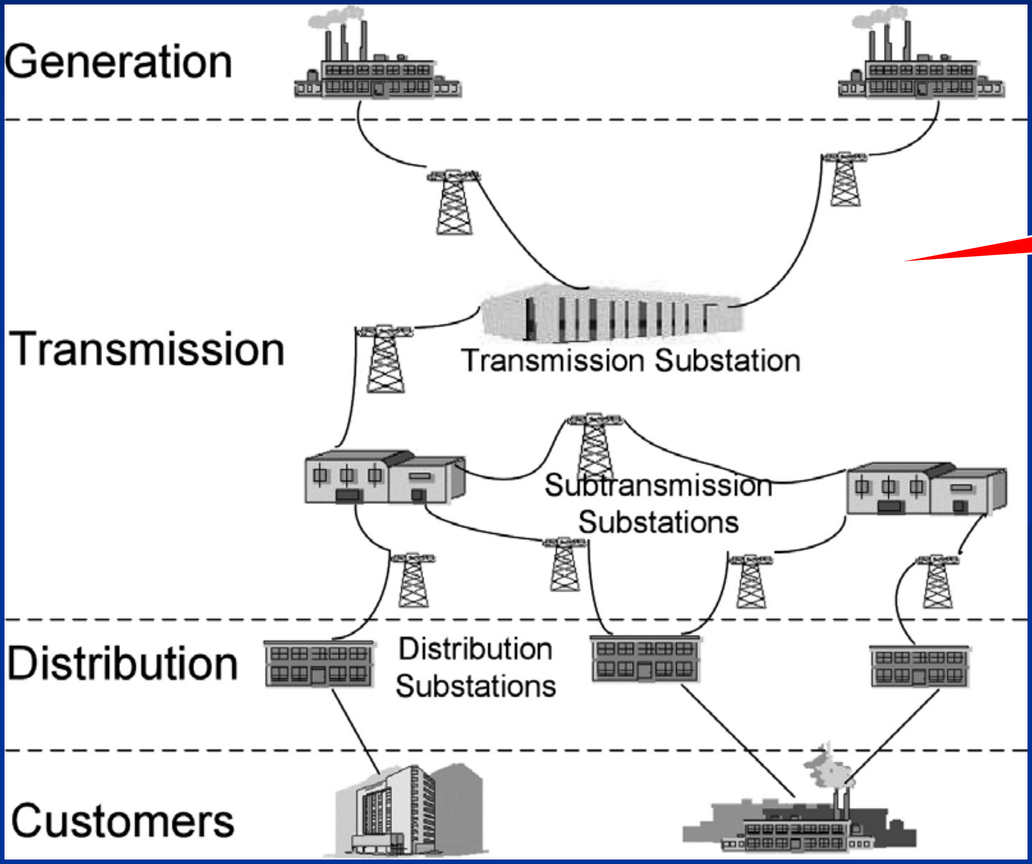
Inter-Area Oscillations

- Power flow at a distance u is :

$$p(u,t) = -\frac{1}{\gamma}\frac{\partial \delta(u,t)}{\partial u}$$



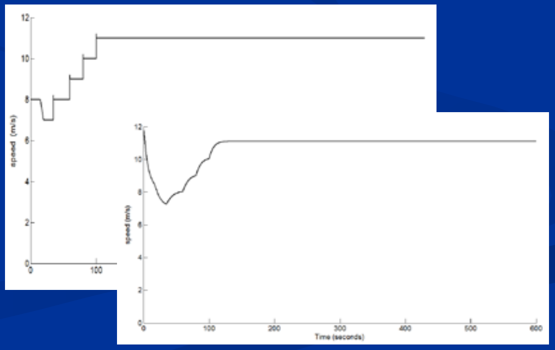
Cyber Security of Electric Power Grids



False Data Injection

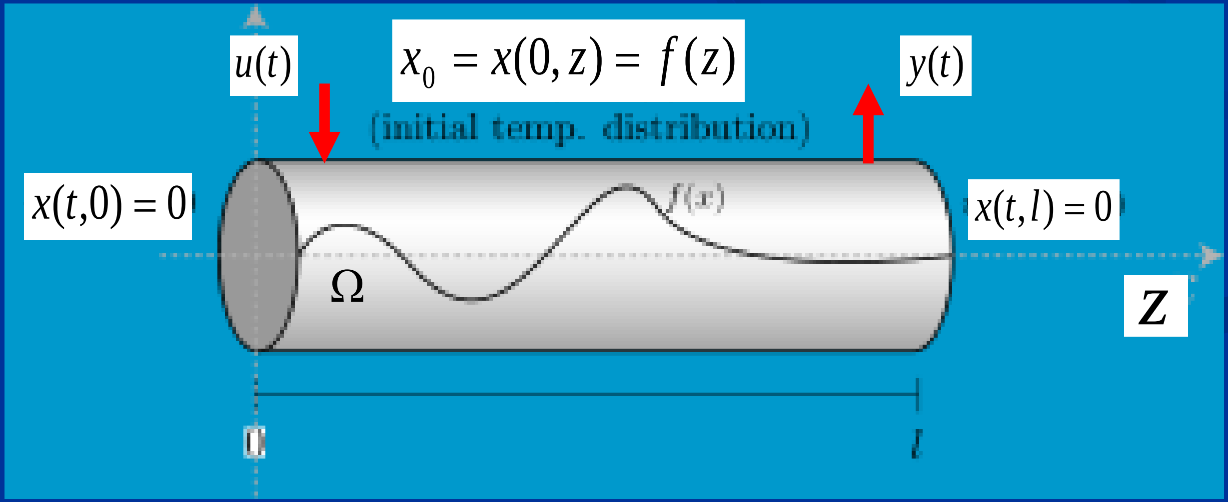
Adaptive Disturbance Tracking Control for Large Horizontal Axis Wind Turbines with Disturbance Estimator in Region II Operation

Mark J. Balas. Kaman S. Thapa Magar *and* Qian Li
ASME 2014



Universal Infinite Dimensional Example: Heat Diffusion

$$
\begin{cases}
\dfrac{\partial x}{\partial t} = \underbrace{\dfrac{\partial^2 x}{\partial z^2}}_{Ax} + bu; \\
b(z) \in D(A) \equiv \left\{ x \,/\, \text{smooth and BC:} x(t,0) = x(t,l) = 0 \right\} \\
\qquad \subset X \equiv L^2(\Omega) \\
\text{with } \left(x, y \right) \equiv \int_\Omega x(t)\, y(t)\, dt \\
x(0) = x_0 \in D(A) \\
y = (c, x); \quad c(z) \in D(A)
\end{cases}
$$



Symmetric Hyperbolic Systems

$$\frac{\partial \underline{\varphi}}{\partial t} = \underbrace{\sum_{i=1}^{n} \underbrace{A_i}_{\substack{lxl \text{ constant} \\ \text{symmetric}}} \frac{\partial \underline{\varphi}}{\partial z_i} + \underbrace{A_0}_{lxl \text{ constant}} \underline{\varphi}}_{A\varphi} ; \underline{x} \in D(A) \subset X \equiv L^2(\Omega; \Re^l)$$

Boundary

$$Conditions: \Lambda(z)\varphi(z,t) = 0 \forall z \in \partial\Omega; t \geq 0$$

$$\text{2-dim wave equation } \frac{\partial^2 x}{\partial^2 t} = \underbrace{\left(\frac{\partial^2 x}{\partial^2 z_1} + \frac{\partial^2 x}{\partial^2 z_2}\right)}_{\Delta x} + \gamma x$$

$$\Leftrightarrow \underline{x}_t = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{A_1} \frac{\partial \underline{x}}{\partial z_1} + \underbrace{\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{A_2} \frac{\partial \underline{x}}{\partial z_2} + \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & \gamma & 0 \end{bmatrix}}_{A_0} \underline{x} \quad \text{where} \quad \underline{x} \equiv \begin{bmatrix} x_{z_1} \\ x_{z_2} \\ x \\ x_t \end{bmatrix}$$

Smart Grid: Inter-Area Oscillations

$$: x_{tt} = v^2 x_{zz} - \eta x_t$$

$$\Leftrightarrow \underline{x} \equiv \begin{bmatrix} x_z \\ x_t \end{bmatrix} \Rightarrow \underline{x}_t = \underbrace{\begin{bmatrix} 0 & v \\ v & 0 \end{bmatrix}}_{A_1} \underline{x}_z + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\eta \end{bmatrix}}_{A_0} \underline{x} \equiv A\underline{x}$$

$$\text{Dirac Equation: } \frac{\partial \phi}{\partial t} = -c\Big(\sum_{i=1}^{3} \underbrace{A_i}_{\substack{Pauli \\ Spin \\ Matrices}} \frac{\partial \phi}{\partial x_i}\Big) + \Big(i\frac{mc^2}{\hbar} I_4\Big)\phi$$

"Simplicity" via Infinite Dimensional Spaces

$$
\begin{cases}
\dfrac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^{m} b_i u_i \\
x(0) = x_0 \in D(A) \subset X \qquad \Rightarrow x(t, w_0) = \underbrace{U(t)x_0}_{\text{Evolution in } X}; \forall t \ge 0 \\
y = Cx = \left[(c_1, x) \quad (c_2, x) \quad \ldots \quad (c_m, x) \right]^T
\end{cases}
$$

"Boil Away" all the special properties of $\Re^N$

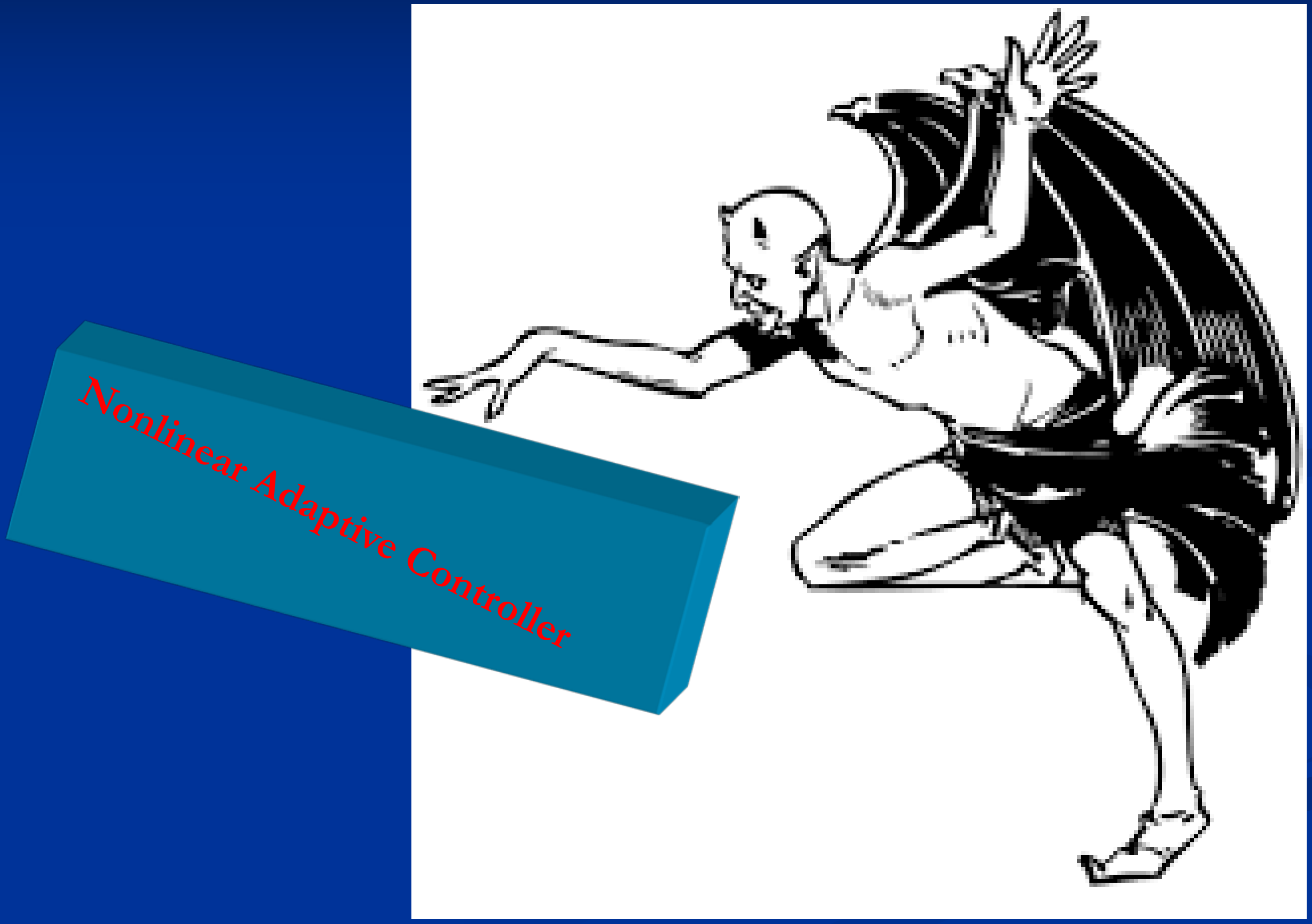
J. Wen & M.Balas, "Robust Adaptive Control in Hilbert Space ", J. Mathematical. Analysis and Applications, Vol 143, pp 1-26,1989.

J. Wen & M.Balas ,"Direct Model Reference Adaptive Control in Infinite-Dimensional Hilbert Space," Chapter in Applications of Adaptive Control Theory, Vol.11, K. S. Narendra, Ed., Academic Press, 1987

C_0 – Semigroup of Bounded Operators $U(t)$:

$$
\begin{cases}
U(t+s) = U(t)U(s) \ \ \text{(semigroup property)} \\
\dfrac{d}{dt}U(t) = AU(t) = U(t)A \ (\ A \text{ generates } U(t)) \\
U(t)x_0 \xrightarrow[t \to 0]{} x_0 \ (\text{ continuous at t} = 0)
\end{cases}
$$

The Devil Lurks in the Details



Semigroups

Closed Linear Operator

Solve $\begin{cases} \dfrac{\partial x}{\partial t} = Ax \\ x(0) = x_0 \in D(A) \end{cases} \Rightarrow x(t) = U(t)x_0$

$\dim X < \infty \Rightarrow U(t) = e^{At} \equiv \sum_{k=0}^{\infty} A^k \dfrac{t^k}{k!}$

C_0 – Semigroup

$U(t) : X \to X$ bounded operators $t \geq 0$

Generator : $Ax = \lim_{t \to 0+} \dfrac{U(t)x - x}{t}$ with $D(A) \equiv \{x / \lim_{t \to 0+}$ exists $\}$ dense in X

LaPlace Transform $\begin{cases} L(U(t)) = (\lambda I - A)^{-1} \equiv R(\lambda, A) \text{ Resolvent Operator} \\ L^{-1}(R(\lambda, A)) = U(t) \end{cases}$

Spectrum of A

$$\text{Resolvent Set } \rho(A) \equiv \left\{ \lambda \,/\, R(\lambda, A) : X \to X \text{ bounded linear op on } X \right\}$$

$$\text{Spectrum } \sigma(A) \equiv \rho(A)^{\mathrm{C}} = \sigma_{\text{point}}(A) \cup \sigma_{cont}(A) \cup \sigma_{residual}(A)$$

$$\sigma_{point}(A) \equiv \left\{ \lambda \,/\, \lambda I - A \text{ is NOT 1-1} \right\} = \left\{ \lambda \,/\, \exists \phi \neq 0 \ni \lambda\phi = A\phi \right\}$$

$$\sigma_{cont}(A) \equiv \left\{ \lambda \,/\, \lambda I - A \text{ is 1-1, but its range is only dense in } X \right\}$$

$$\sigma_{residual}(A) \equiv \left\{ \lambda \,/\, \lambda I - A \text{ is 1-1, but range is a proper subspace of } X \right\}$$

When is a Semigroup Exponentially Stable ?

Lumer – Phillips (Renardy & Rogers 1993) :

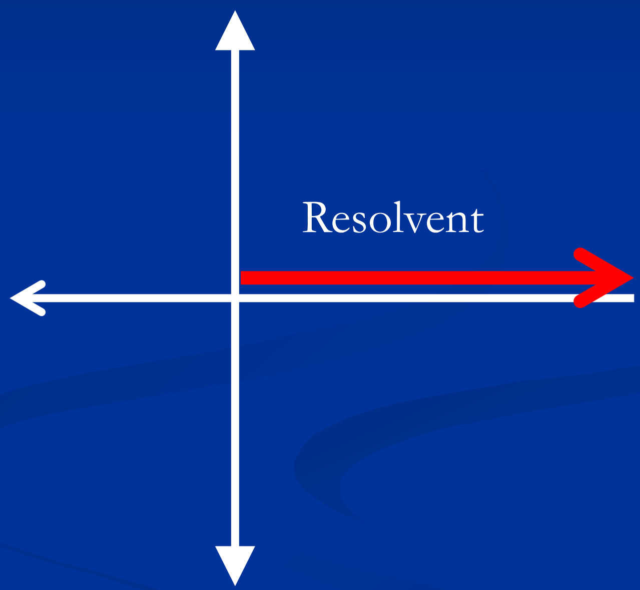
$D(A)$ dense in complex Hilbert space X,

$\mathrm{Re}(Ax, x) \leq -\alpha \|x\|^2 \; \forall x \in D(A), \alpha > 0$

and

$\exists$ real $\lambda_* > -\alpha \ni A - \lambda_* I$ is onto

$\Rightarrow \|U(t)\| \leq e^{-\alpha t}; t \geq 0$ (exponentially stable semigroup)



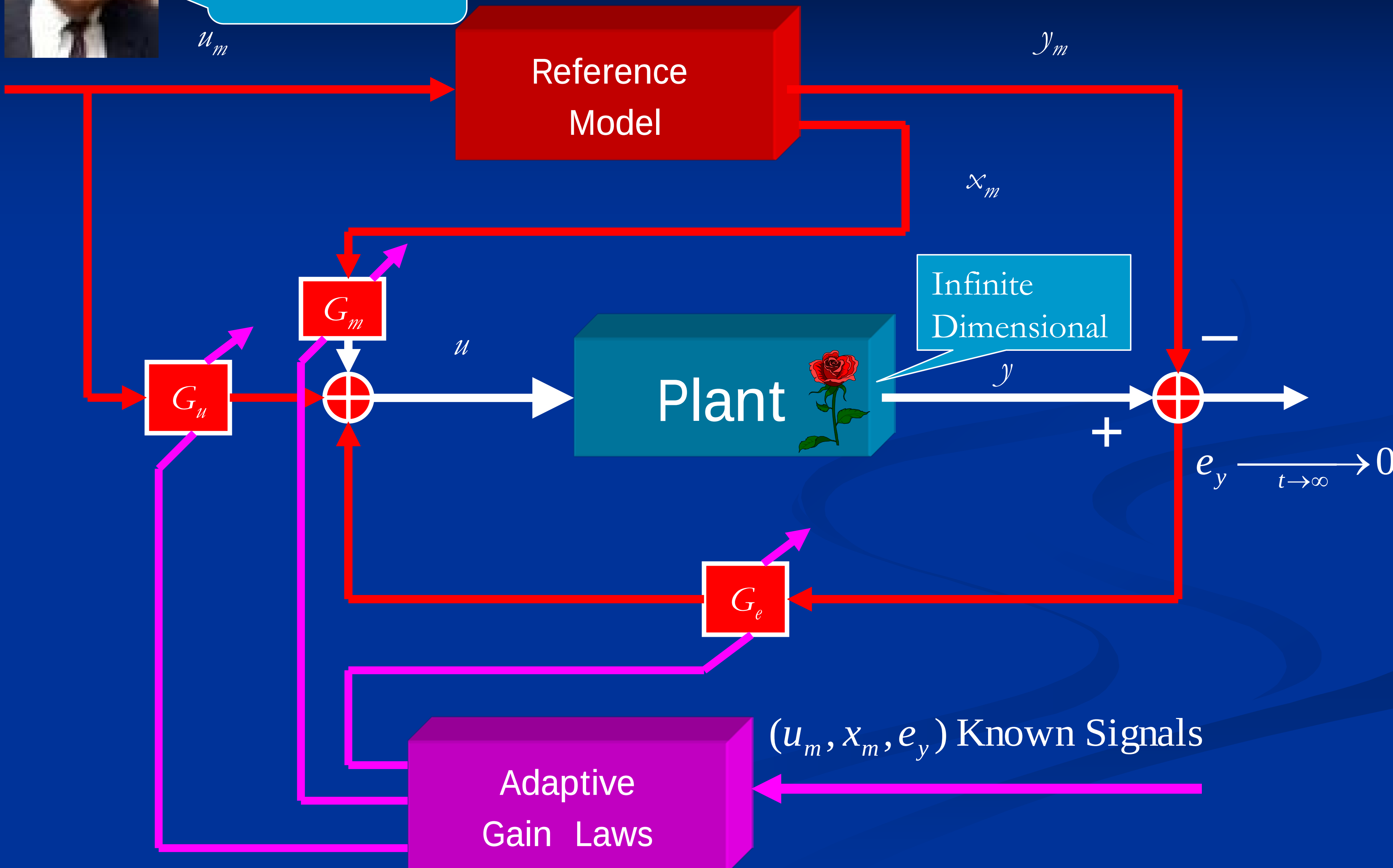
Theorem (Gearhart, Pruss, & Greiner) :

Assume A generates a C_0 - semigp $U(t)$ on a <u>Hilbert</u> space X.

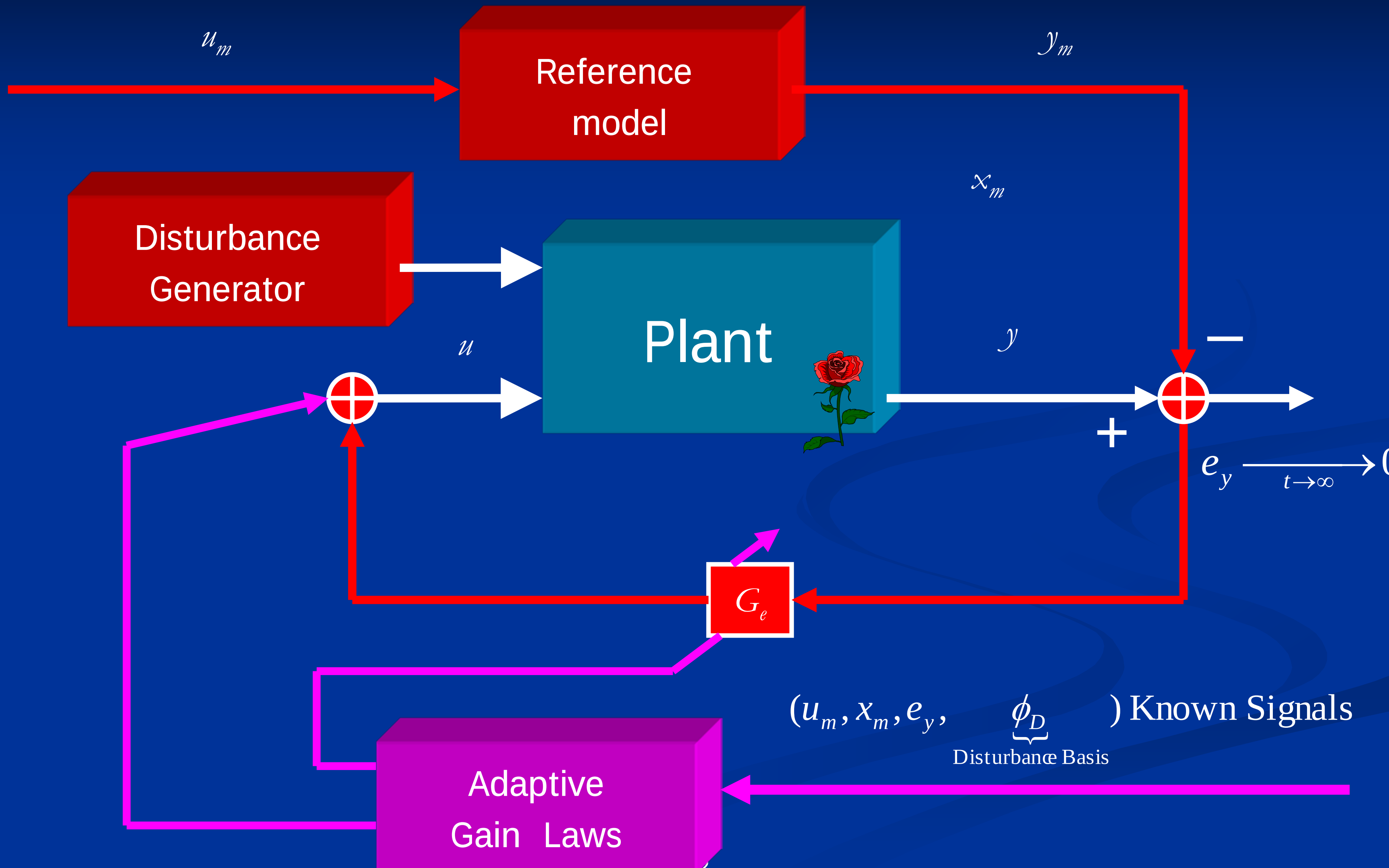
$U(t)$ is exponentially stable $\Leftrightarrow \mathrm{Re}\lambda > 0 \Rightarrow \lambda \in \rho(A)$ and

$\|R(\lambda, A)\| \leq M < \infty$, for <u>all</u> such complex λ

Direct Adaptive Model Following Control (Wen-Balas 1989)



Direct Adaptive Persistent Disturbance Rejection (Fuentes-Balas 2000)



Stability via Lyapunov-Barbalat

Nonlinear Dynamics $\begin{cases} \dot{x} = f(t,x) \\ x(0) = x_0 \in \Re^N \end{cases}$

Find Energy-like Function : $V(x)$

$V(x) > 0$ when $x \neq 0$

$V(0) = 0$

~~$\dot{V} = gradV * f(t,x) < 0 \Rightarrow x(t) \to 0$ as $t \to \infty$ for all x_0~~

Often does
Not happen

$\dot{V} \leq 0 \Rightarrow$ All trajectories $x(t)$ are bounded

From Barbalat's lemma :

$\dot{V}(t) \leq 0$ and uniformly continuous $\Rightarrow \dot{V}(t) \to 0$ as $t \to \infty$

Infinite-Dimensional Lyapunov-Barbalat Theory: PDE & Delay Systems

X Hilbert or Banach Space

Lyapunov-Balas

$$\text{Let}\begin{cases} V(t,x,\Delta G)\equiv V(t,x)+\dfrac{1}{2}tr(\Delta G\gamma^{-1}\Delta G^{T}) \\ \text{with } x(t)=U(t)x_0\in X;t\geq 0 \end{cases}$$

Linear or Nonlinear Evolution

$$\text{Theorem: If}\begin{cases} \alpha(\|(x,\Delta G)\|)\leq V(t,x,\Delta G)\leq \beta(\|(x,\Delta G)\|) \\ \dot{V}(t,x,\Delta G)\leq -W(x)\leq 0 \end{cases}$$

and $\dfrac{dW(x(t))}{dt}=(\underbrace{\dfrac{\partial W}{\partial x}}_{\substack{Frechet\\ Derivarive}})\dfrac{\partial x(t)}{\partial t}$ is bounded, then $W(x(t))\xrightarrow[t\to\infty]{}0$ and ΔG bounded.

If *W(x)* is <u>coercive</u> in the partial state x ,or $W(x)\geq\gamma(\|x\|)$, then $x(t)\xrightarrow[t\to\infty]{}0.$

Linear System Strict Dissipativity

Energy Storage Function : $\begin{cases} V(x) \equiv (x, Px) > 0; \forall x \neq 0 \\ V(0) = 0 \end{cases}$

A Linear Dynamic Infinite-Dimensional System is STRICTLY DISSIPATIVE when

$$\exists P : X \xrightarrow[]{\substack{Linear\ Op \\ Self-Adjoint \\ Positive}} X$$

$$p_{\min} \|x\|^2 \leq V(x) \equiv (Px, x) \leq p_{\max} \|x\|^2 \ni$$

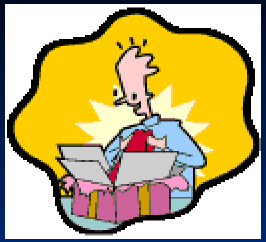
$$\begin{cases} \mathrm{Re}(PAx, x) \equiv \frac{1}{2}[(PAx, x) + (x, PAx)] \leq -\underbrace{\alpha \|x\|^2}_{W(x)}; \forall x \in D(A) \\ PB = C^* \end{cases}$$

When $P \equiv I \Rightarrow$ Lumer - Phillips

DISSIPATIVE when $\alpha = 0$

$$\Rightarrow \frac{1}{2} \underbrace{\frac{dV}{dt}}_{\substack{Energy \\ Storage \\ Rate}} = \underbrace{\mathrm{Re}(Px, x)}_{\leq -\alpha\|x\|^2} + \underbrace{(x, PBu)}_{(y,u)} \leq \underbrace{(y, u)}_{\substack{External \\ Power}} - \underbrace{\alpha \|x\|^2}_{\substack{Internally \\ Dissipated \\ Power}}$$

For Finite & Infinite Dimensions
All Roads Lead To Rome



Control Porno

$$\begin{cases} \dfrac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^{m} b_i u_i \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = [(c_1, x) \quad (c_2, x) \quad \dots \quad (c_m, x)]^T \end{cases}$$

with (A, B, C) Almost Strictly Dissipative (ASD)

$$\Rightarrow \textit{Direct}\text{ Adaptive Controller } \begin{cases} u = Gy \\ \dot{G} = -yy^* \sigma; \sigma > 0 \end{cases}$$

produces $x(t) \xrightarrow[t \to \infty]{} 0$

with bounded adaptive gains $G(t)$

Finite- Dimensional LINEAR ASD: Two Simple Open-Loop Properties

High Frequency Gain is Sign-Definite (CB>0)

Open-Loop Transfer Function is Minimum Phase (i.e. Transmission Zeros are all stable)

⟺ **Almost Strictly Dissipative**

⟹ Adaptive Regulation $\begin{cases} u = Gy \\ \dot{G} = -yy^{*}\sigma; \sigma > 0 \end{cases}$

produces $x(t) \xrightarrow[t\to\infty]{} 0$

with bounded adaptive gains $G(t)$

Transmission Zeros: Infinite Dimensional Systems

For $\begin{cases} \dfrac{\partial x}{\partial t} = Ax + Bu \\ y = Cx; \;\; x(0) = x_0 \end{cases}$

λ_* is a $\underline{\text{transmission (or transmission - blocking) zero}}$

of (A, B, C) when,

for $x_0 \equiv 0, \;\; u = e^{\lambda_* t} w \Rightarrow y \equiv 0$

λ_* is a transmission zero

of (A, B, C) when

when $N(H(\lambda_*)) \neq \{0\}$

where $H(\lambda) \equiv \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix}$:

$D(A) x \Re^M \rightarrow X x \Re^M$

closed linear operator

Normal Form

$$\underline{CB \text{ nonsingular}} \Rightarrow X = \underbrace{R(B)}_{sp\{b_1,b_2,\ldots,b_m\}} \oplus \underbrace{N(C)}_{\substack{sp\{\theta_1,\theta_2,\ldots\} \\ \text{orthomormal}}}$$

Via Non-Orthogonal Projections

$P_1 \equiv B(CB)^{-1}C$ onto $R(B)$

$P_2 \equiv I - P_2$ onto $N(C)$

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \underbrace{\Rightarrow}_{CB \text{ nonsing}}$$

$\underline{\text{Normal Form}}$: $y \in \Re^{M}$ & $z \in l_2$

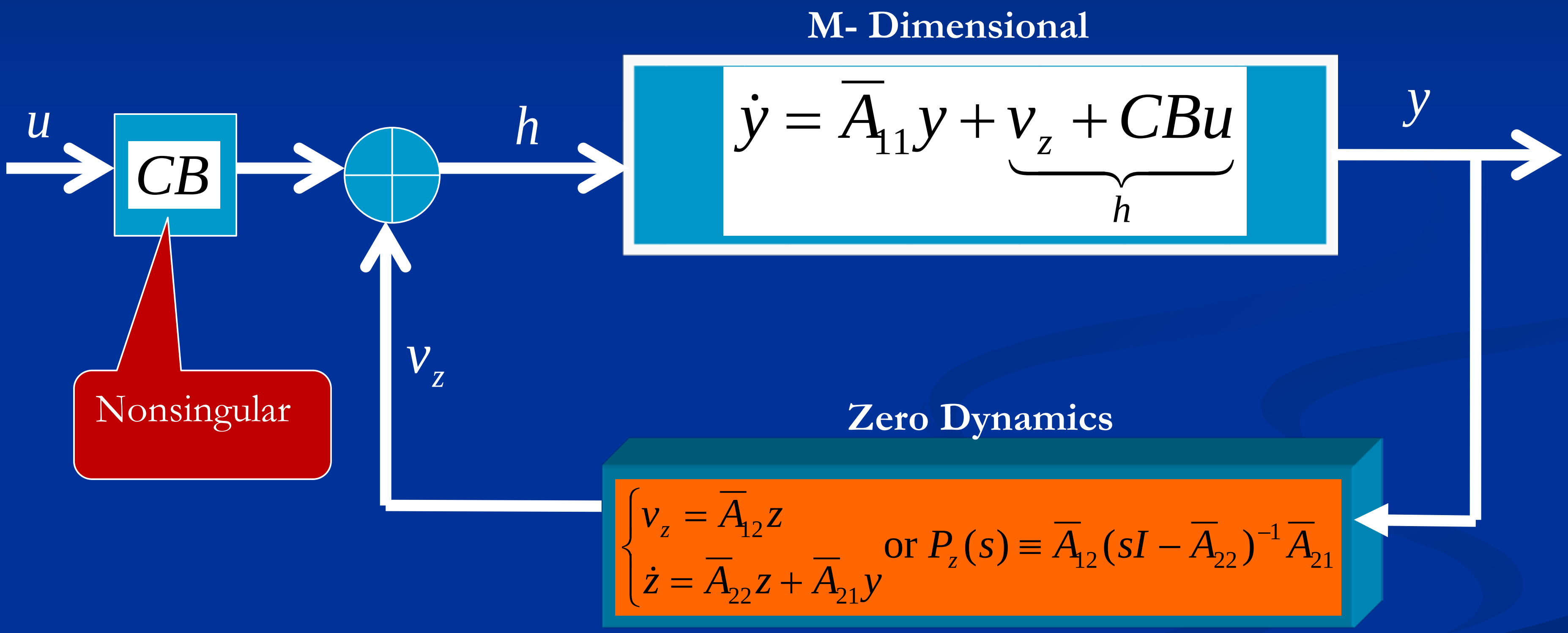
$$\begin{cases} \dot{y} = \bar{A}_{11}y + \bar{A}_{12}z + CBu \\ \dot{z} = \bar{A}_{21}y + \bar{A}_{22}z \end{cases} \Leftrightarrow \begin{cases} \begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} + \begin{bmatrix} CB \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} I_m & 0 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} \end{cases}$$

$(\bar{A}_{22}, \bar{A}_{21}, \bar{A}_{12})$ called the "zero dynamics"

$$\text{Result: } \underbrace{Z(A,B,C)}_{\text{Transmission Zeros}} = \sigma(\bar{A}_{22})$$

Also (A,B,C) ASD if and only if Normal Form is ASD

Zero Dynamics (Normal Form)



Note: Zero Dynamics are Invariant under Output Feedback

My Infinite-Dimensional Version of the "Two Simple Open Loop Properties" Theorem

$$\begin{cases} \dfrac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^{m} b_i u_i ; A \text{ generates a } C_0 \text{ semigroup} \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = \left[(c_1, x) \quad (c_2, x) \quad \ldots \quad (c_m, x)\right]^* ; b_i , c_j \in D(A) \end{cases}$$

Pretty Close !!

Theorem: Def : $\lambda_* \in C$ is a transmission zero of (A,B,C) when $N(H(\lambda_*)) \neq \{0\}$

where $H(\lambda) \equiv \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} : D(A)x\Re^M \to Xx\Re^M$ closed linear operator

(A,B,C) is Almost Strictly Dissipative if and only if

$CB = \left[(c_j, b_i)\right]_{\text{mxm}} > 0$ and Transmission Zeros$(A,B,C) \equiv \{\lambda / N(H(\lambda)) \neq \{0\}\} = \sigma_p(\overline{A}_{22})$ "*stable*"

(i.e., $\overline{A_{22}}$ generates exponentially stable semigroup)

Adaptive Augmentation

Autonomic System !!!

Nonlinear System

$$\begin{cases} \dot{x} = A(x) + B(x)u + \Gamma u_D \\ y = C(x) \end{cases}$$

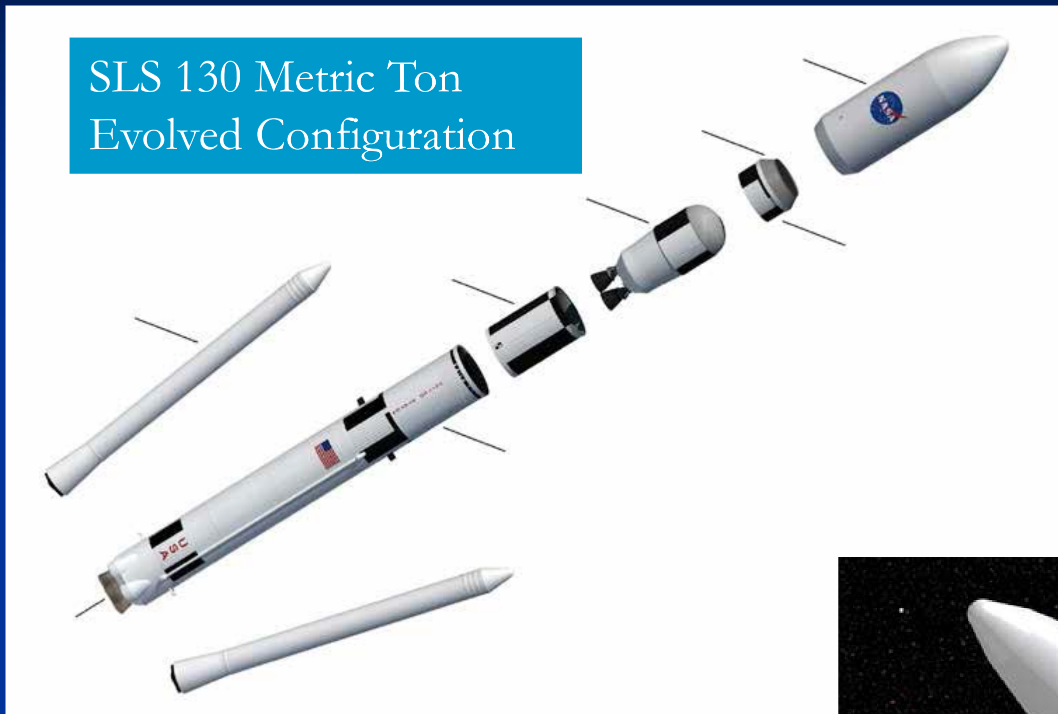
Fixed Gain Controller

$$\begin{cases} u = L_{11}y + L_{12}\zeta \\ \dot{\zeta} = L_{21}y + L_{22}\zeta \end{cases}$$

Adaptive Regulator

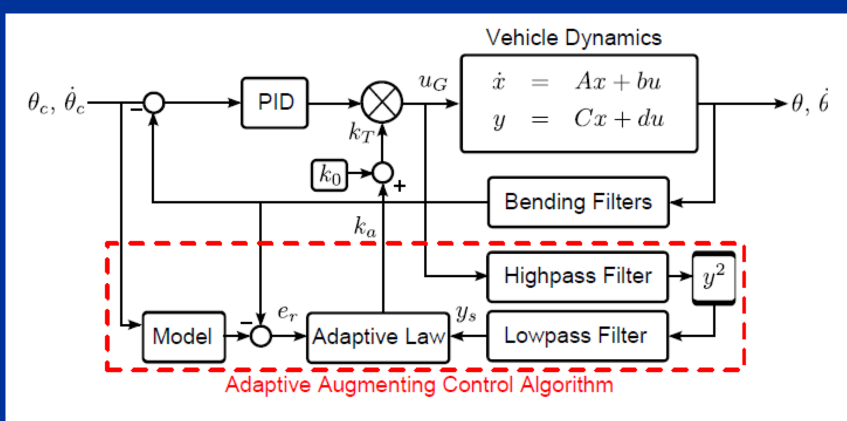
$$\begin{cases} u = G_e y \\ \dot{G}_e = -[yy^T]\gamma ; \gamma > 0 \end{cases}$$

NASA Space Launch System SLS



Adaptive Control
=Risk Management

NASA MSFC

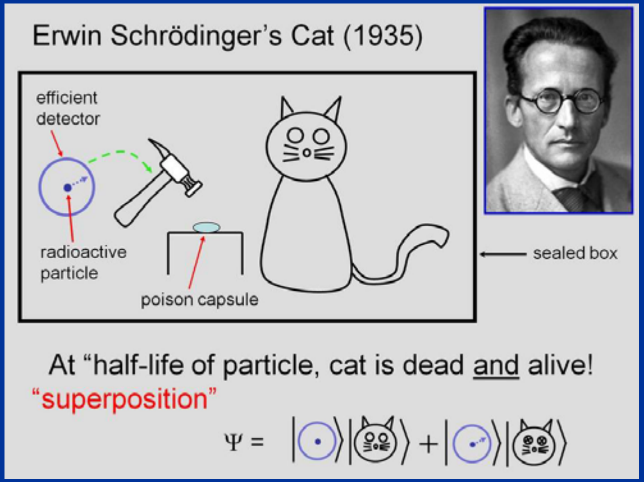


Adaptive Control in Quantum Information Systems

This might be the most fundamental application of direct adaptive control

Quantum Computing

A Quantum computer will operate differently from a Classical one.
It will be involved w physical systems on an atomic scale,
eg atoms, photons, trapped ions, or nuclear magnetic moments



Unitary → Reversible

Could be improved with Adaptive Control
So that Quantum Error Correctiion can work!!!

Quantum Basics (Dirac & Von Neumann)

Observable $A : X \xrightarrow[\text{self-adjoint}]{\text{bounded}} X$

Compact Resolvent $\Rightarrow Ax = \sum_{k=1}^{\infty} \lambda_k \underbrace{(x, \varphi_k)\varphi_k}_{P_k x}$

Orthonormal Eigen – Basis for X

Pure States : φ_k eigenfunctions of A

Mixed State $\varphi \in X$ complex Hilbert Space :

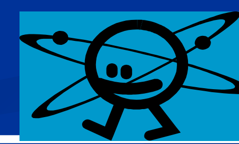
$$(\varphi, \varphi) = 1 \text{ or } \|\varphi\| = 1 \Rightarrow \varphi = \sum_{k=1}^{\infty} c_k \varphi_k \;\&\; 1 = \|\varphi\|^2 = \sum_{k=1}^{\infty} |c_k|^2$$

$\therefore$ "A mixed state is a linear combination of pure states"

Schrodinger Wave Equation

$$i\hbar \frac{\partial \varphi}{\partial t} = \underbrace{H_0}_{\substack{\text{Skew Self}-\text{Adjoint} \\ \text{Compact} \\ \text{Resolvent}}} \varphi + H_C(u)\varphi$$

$\therefore$ Discrete Spectrum $\sigma(H_0) = \{i\lambda_k\}_{k=1}^{\infty}$



$-\infty$

Marginally Stable

$\Rightarrow U_0(t) : X \to X$ Unitary Group (reversible)

and $U_0(t)\varphi = \sum_{k=1}^{\infty} e^{i\lambda_k t} \langle \varphi, \phi_k \rangle \phi_k$ with $\langle \phi_k, \phi_l \rangle = \delta_{kl}$

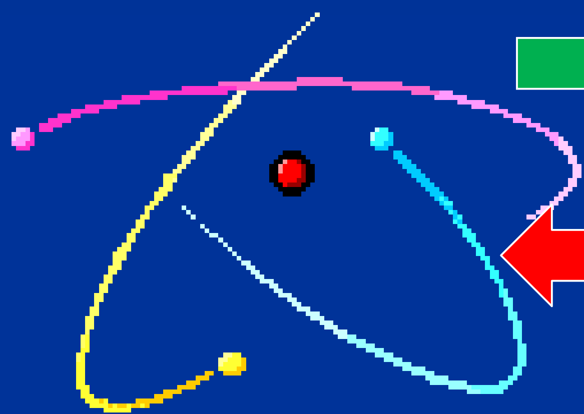
Quantum Measurement

Entanglement

$$X = X_S \otimes X_M$$

$$\varphi = \sum_{k,l} \alpha_{kl} (\varphi_k^S \otimes \varphi_l^M) \neq h \otimes w$$

S



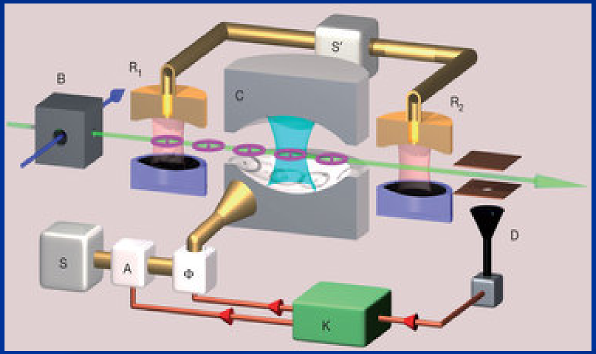
M

Back Action

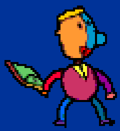
Ontology (what is) vs Epistemology (What is measured)
Existence =Interaction

Small Quantum Systems

- We <u>can</u> begin to experiment with just one electron, atom or small molecule



- Need:



Precise control

Isolation from the environment

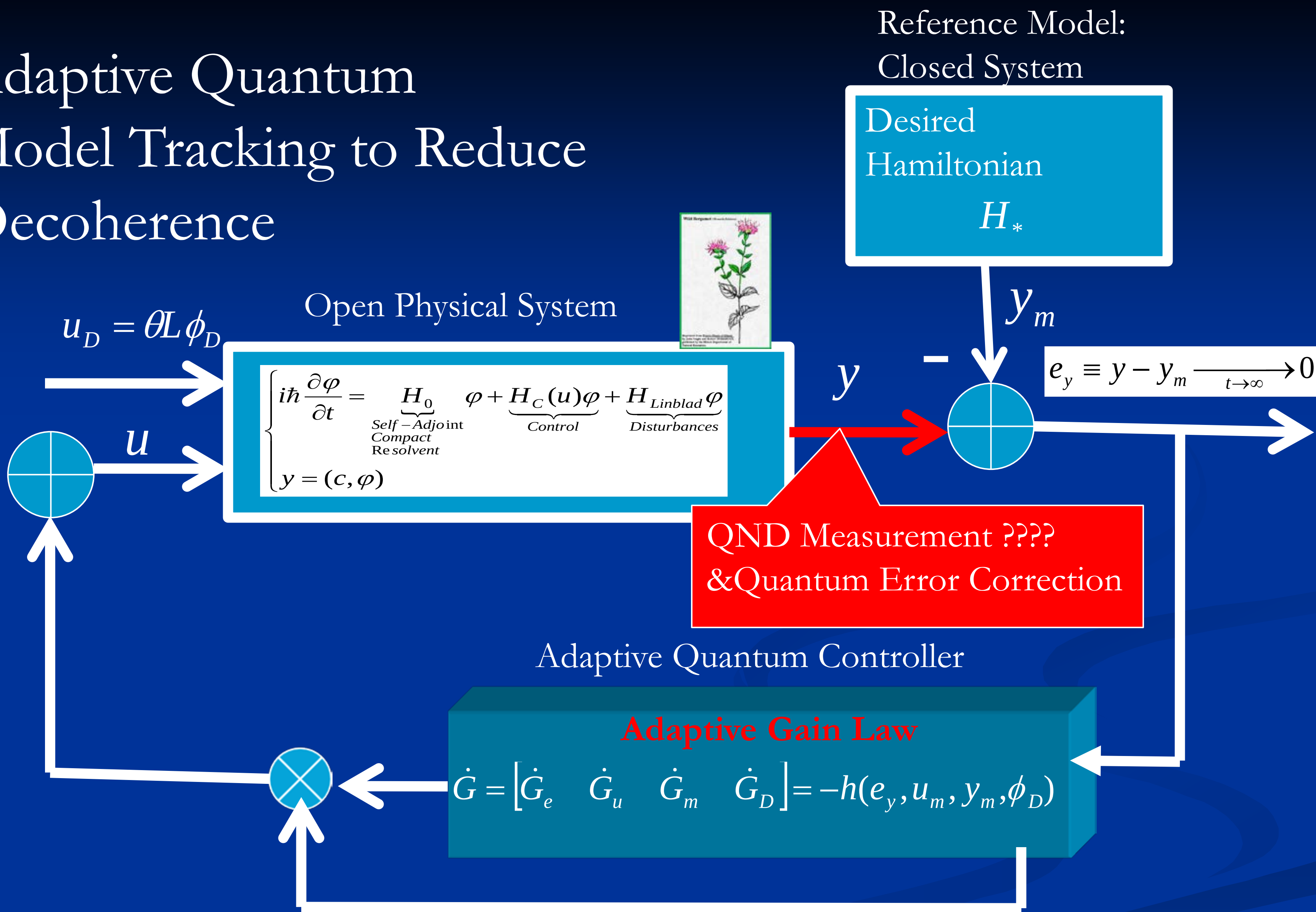
Simple small systems : single particles or small groups of particles

…… David Wineland NIST

Physics Nobel Prize 2012

S. Haroche & D. Wineland

Adaptive Quantum Model Tracking to Reduce Decoherence



Famous
Lisbon Poet

“No intelligent idea can gain general acceptance unless some stupidity is mixed in with it” …..

Fernando Pessoa, The Book of Disquiet